The Use of the Rule of Proportion IN ARITHMETICK AND Geometrie.

First published at *Paris* in the *French-tongue*, and dedicated to *Monsieur*, the then Kings onely Brother (now Duke of *Orleance*)

By *Edm. Wingate*, an English *Gent*.

And now translated into *English* by the same Author.

Whereinto is now also inserted the *Constructi-on* of the same Rule, and a farther use thereof, in Questions that concern

Astronomie,	*Gaging of Vessel,*
Dialling,	*Military Orders,*
Geographie,	*Interest and*
Navigation,	*Annuities.*

The second Edition inlarged and amended.

Ecclesiasticus 39. 17.

None may say, what is this? wherefore is that? for at time convenient they shall all be sought out.

LONDON,
Printed by *J. B.* for *Philemon Stephens*, at the Gilded Lion in Pauls-Churchyard, 1658.

A Tres-haut & Tres-puissant Prince,
MONSIEUR, Gaston de France,
Frere unique du Roy, Duc
d'Anjou, &c.

Monseigneur,

QUelque temps apres mon arrivée en ceste ville, ayant fait voir l'Instrument, dont i'explique les utilitez en se liuret, & discouru de quelques uns de ses usages, i'appris de plusieurs, que si l'on travailloit sur ce sujet, que le labour en serroit bien recueilly: Cela (pour vous dire la verité) m'a enhardy d'en dire quelque chose, & (luy faisant voir le jour) me targuer de vostre authorité; Vous me pardonerez toutesfois, si i'ay eu ceste hardiesse de luy donner du credit, & de la recommendation de ce costè la, comme i'ay eu la voluntè de tesmoigner combien ie suis,

MONSEIGNEUR,

Vostre tres-humble

& tres-obeyssant

serviteur,

Edm. Wingate.

To the High and Mighty Prince, MONSIEUR, Gaston of France, the Kings onely Brother, Duke of Anjou, &c.

MY LORD,

NOt long since after my arrival in this City, having divulged the Instrument (whose uses I explain in this little Treatise) and discoursed of some of the conveniences thereof, I was given to understand by diverse, that if pains were bestowed upon that subject, the labour therein taken might obtain good reception: This (to say truth) hath given me encouragement thereof to say somewhat, and (having caused it to see the light) to shelter it under your protection: Neverthelesse you shall pardon me, for that by presuming to procure unto it from thence credit and recommendation, I have expressed a willingnesse to testifie, how much I am,

MY LORD,

Your most humble and

most obedient

servant,

Edw. Wingate.

THE PREFACE TO THIS TRANSLATION.

AMongſt the many rare effects produced by the noble invention of *Logarithmes*, the projection of the *Rule of Proportion* is not the leaſt, which being firſt diſcovered by that Learned and Induſtrious Artiſt *Edm. Gunter* (late Profeſſor of *Aſtronomy* in Greſham Colledge, London, deceaſed) was by me (in *Anno* 1624.) tranſported into *France*, and there communicated to moſt of the chiefeſt Mathematicians then reſiding in *Paris*, who apprehending the great benefit that might accrew thereby, importuned me to expreſs the uſe thereof in the *French* tongue; which being performed accordingly, I was adviſed by M^r^ *Alleaume* (the Kings chief Ingenier) to dedicate my Book to *Monſieur*, the then Kings onely Brother, now Duke of *Orleans* : Neverthelesſ this Work (as it was there publiſhed) coming forth as an *Abortive*, (the publiſhing thereof being ſomewhat haſtned, by reaſon an Advocate of *Dijon* in Burgundy began to print ſome uſes thereof, which I had in a friendly way communicated unto him) I thought it not worthy to ſee the light here in *England*, eſpecially in regard M^r^ *Gunter* himſelf had learnedly explained the uſe thereof in a far larger volume: Howbeit having now of late (by reaſon of the preſent troubles) had too much leiſure from my other employments and calling, to look back to thoſe ſtudies, wherewith in my younger time I uſed to buſie my ſelf; And having alſo upon that occaſion bethought my

To the High and Mighty Prince, MONSIEUR, Gaſton of France, the Kings onely Brother, Duke of Anjou, &c.

MY LORD,

NOt long ſince after my arrival in this City, having divulged the Inſtrument (whoſe uſes I explain in this little Treatiſe) and diſcourſed of ſome of the conveniences thereof, I was given to underſtand by diverſe, that if pains were beſtowed upon that ſubject, the labour therein taken might obtain good reception: This (to ſay truth) hath given me encouragement thereof to ſay ſomewhat, and (having cauſed it to ſee the light) to ſhelter it under your protection: Nevertheleſſe you ſhall pardon me, for that by preſuming to procure unto it from thence credit and recommendation, I have expreſſed a willingneſſe to teſtifie, how much I am,

MY LORD,

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AMongst the many rare effects produced by the noble invention of *Logarithmes*, the projection of the *Rule* of *Proportion* is not the least, which being first discovered by that Learned and Industrious Artist *Edm. Gunter* (late Professor of *Astronomy* in Gresham Colledge, London, deceased) was by me (in *Anno* 1624.) transported into *France*, and there communicated to most of the chiefest Mathematicians then residing in *Paris*, who apprehending the great benefit that might accrew thereby, importuned me to express the use thereof in the *French* tongue; which being performed accordingly, I was advised by Mr *Allcaume* (the Kings chief Ingenier) to dedicate my Book to *Monsieur*, the then Kings onely Brother, now Duke of *Orleans* : Neverthelesss this Work (as it was there published) coming forth as an *Abortive*, (the publishing thereof being somewhat hastned, by reason an Advocate of *Dijon* in Burgundy began to print some uses thereof, which I had in a friendly way communicated unto him) I thought it not worthy to see the light here in *England*, especially in regard Mr *Gunter* himself had learnedly explained the use thereof in a far larger volume: Howbeit having now of late (by reason of the present troubles) had too much leisure from my other employments and calling, to look back to those studies, wherewith in my younger time I used to busie my self; And having also upon that occasion bethought my

 self

ſelf, how diverſe neceſſary additions might be fitly inſerted into that Work, and many inconveniences in the uſe of that *Inſtrument*, which before did uſually incumber the Practitioner might be removed; I have adventured to let this Tranſlation appear; In which you ſhall find expreſſed, as ſuccinctly and plainly as I could; the uſe of that *Rule* in the form as you find it annexed to this Book: Not that I would confine any man to uſe ſuch a form and no other, but becauſe the operations are thereupon underſtood and performed more perſpicuouſly and plainly, then (as I conceive) they would be, if the lines were thereupon otherwiſe deſcribed; Howbeit the uſe thereof being in this form once gained, the Practitioner may then uſe that way of deſcribing it, which ſorts beſt with his own humor.

Having thus acquainted you with the occaſion of publiſhing this Treatiſe, leaſt I may now expoſe it to prejudice, give me leave to premiſe theſe few advertiſements following: Firſt, therefore, it is deſired that he, who intends to read this book with profit, ſhould have a proper *Genius* and Phanſie for the *Mathematicks*, not onely ready to conceive *Mathematical* notions; but likewiſe able to wreſtle with them, and apt to take pleaſure in them: For *De quolibet ligno non fit Mercurius*. Again, it is expected he ſhould be aforehand furniſhed with competent knowledge in thoſe Sciences, *viz.* 1. In *Arithmetick* he ought to be acquainted with the nature of numbers, whole and broken, abſolute and relative; with Numeration, Addition, Subſtraction, Multiplication, Diviſion, the Rule of three, direct and inverſe; with the nature and extraction of Roots, ſquare and cube; and with the right uſe of *Logarithmes*. 2. In *Geometrie*, to be verſt in the doctrine of Triangles, plain & ſpherical, & (in ſome competent meaſure) to know their nature, together with the w[illegible] & reaſon of their dimenſion; as alſo the dimenſion of other Geometrical figures. 3. In *Aſtronomy*, *Dialling* and *Geographie*, to underſtand that the Problemes which conc[illegible]

the

them, are resolved by the particular application of the doctrine of spherical Triangles to those several sciences. 4. In *Navigation*, to be indifferently well read in such Authors as have explained that Art, and to be able therein also to make use of the doctrine of Triangles: With the knowledge of these things (I say) and the like he ought to be (in some reasonable sort) supplied, that intends to make a right and compleat use of this Treatise: For, none (I presume) will expect to find an intire body of the *Mathematicks* in this small bulk, which is onely intended for an *Enchiridion* or Manual of such Mathematical Rules and *Analogies* as may most properly serve for the resolution of Problemes, which may be wrought upon this *Instrument*: And therefore I wholly refer the *Reader* for demonstrations and larger explanations of the matters in this book contained, to the further scrutiny of other Authors; not doubting but that (upon due perusal hereof) he will find as much inserted, as shall bee thought necessary to discover the manifold and exquisite use of the same *Instrument*. But here I would not bee mistaken, as if I did totally exclude all others, who are not prepared with such an universal knowledge in the *Mathematicks*, from having any capacity at all of understanding this book; For, if he be onely in part acquainted with some of the above-mentioned Learning, he may be able to make use of this *Instrument* according to that degree of knowledge which he hath therein: For example, if he onely know *Multiplication* and *Division*, this Treatise will instruct him how to *multiply* and *divide* upon the *Rule*; and so in like sort of the rest: Howbeit (as I said before) if he intend to have an intire understanding of the uses of this *Instrument*, he must be also furnished with an intire knowledge of all the *Mathematicks*; because it is subservient to every branch of those sciences: And then the conveniency thereof wil have such latitude, that it will not be confined to those uses onely promised in the Title of this Book, but likewise (by the variety

of Rules and Examples therein found) may be readily and fitly applied to other Arts and Professions not there remembred; As namely, in *Fortification*, the Ingenier may here be taught how to find the sides of his *Polygonical* figures, the *Lines* of fortification according to the Rules of that Art, the *quantity* of Trenches and Ramparts, how to order and estimate the labour and work of Pioners, and the like. The *Surveyor* also may here furnish himself with diverse expeditious dispatches, for the taking of distances, the summing up of Plots, being first divided into triangles, the distribution of Fields or Lordships to several persons, the cutting off any part of a triangle or plot according to any quantity propounded, &c. The like may be said of *Musick*, *Architecture*, the *Perspectives*, *Gunnery*, &c. the *Goldsmith* also, and *Mint-master* may here learn how to temper their Allegations: The *Merchant* and *Tradesman*, how to resolve questions of Partnership, & to cast up the value of their comodities: The *Justice* of *Peace* and *High-Constable*, how to rate a Town, Hundred, or County, &c. All which, and much more, must be wholly left to the discretion of those, that will take the pains to understand the use of the said *Instrument*; which (I perswade my self) no man (affecting the *Mathematicks*) will think much to undergo, considering the benefit he may reap thereby, and the delight he may take therein: For, by help thereof, and of a pair of Compasses, onely six Inches long, he may resolve with requisite exactness any proposition in the Arts and Sciences above-remembred (which comes within the bounds of ordinary practise) without the help of pen or paper, and shall thereby also perform more in one hour, then otherwise (I mean by ordinary *Arithmetick*) he shall be able to dispatch in two whole days.

But it may be objected, if this *Instrument* be of such excellent use as is here pretended, why hath it not been heretofore of greater esteem, it being now above twenty years since it was first invented? This objection may be

answered

answered diverse ways: 1. It is no easie matter to drive men out of their old track, especially when they have entertained an opinion that there can be none better. 2. Again, the use thereof in the point of *Numbring* upon the *Rule* (which ought to be accounted the chiefest, and indeed the ground of all the rest) hath not been heretofore (under favour) so fully explained, as here you shall find it: For albeit (I confesse) it were great presumption in me to assume to my self the reputation of having better abilities to describe any of the uses thereof, then M^r *Gunter* himself had, who first invented it; yet this I can aver upon mine own knowledge, that he did forbear to explain that use thereof, because he took it for granted none would meddle with it, but such onely as were already well able to understand how to number upon it, having before hand acquainted themselves with the manner of *numbring* upon *scales*, and with the nature of *Logarithmes*: For, when after my return out of *France*, I importuned him to make a fuller explanation how to number upon it, to the end the use thereof might by that means be made more publick, his answer was, *That it could not be expected the Rule should speak*: Intimating thereby, that the Practitioner should (in that point) rely much upon discretion, and not altogether depend upon precepts and examples. But lastly, the chiefest causes why this *Instrument* hath been hitherto obscured, and the uses thereof no better known to the world, are these: 1. The Difficulty of describing the lines thereupon with convenient exactness. 2. The trouble of working thereupon by reason (sometimes) of too large an extent of the Compasses. 3. The importablenesse thereof, it being requisite for working upon such a *Rule* (onely two foot long) to use a pair of Compasses of nine Inches: 4. The charge of purchasing such an *Instrument* made of brass or wood; for, none but such have been heretofore used. For remedy of the first of these, I have caused the plate, whereupon this *Instrument* is printed, to be portraicted

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traicted with a great deal of care and circumſpection, ſo that *I* dare affirm it to be as exactly drawn (for the main and moſt conſiderable diviſions thereof) as may be expected from Art: For the ſecond, having there three ſeveral Lines of *Numbers* by degrees one leſſe then another, when the Compaſſes are too little for one, you may uſe another; alſo *Croſſe-work* upon the Greateſt Line, will prevent the too great extenſion of the Compaſſes; ſo that it will be requiſite to uſe wth this *Inſtrument* (as it is now contrived) a pair of Compaſſes onely ſix Inches long, as I ſaid before; and yet the diviſions of this (I mean upon the great Line of *Numbers*) are near as large again, as thoſe upon Mr *Gunter's* Rule of the like length: The third and fourth impediments may alſo be remedied, if inſtead of braſs or wood you uſe the impreſſion of the ſaid Plate upon Vellum or Imperial paper, which may either be rolled up and couched in a little box, or otherwiſe paſted upon a Ruler, either flat, to uſe at home, or round, to be carried in a hollow Staffe or Cane, together with the Compaſſes which are to be uſed therewith. Alſo diverſe uſeful conveniences ſhall you meet withall in this Edition of the *Rule*; as namely, a readier way of finding out *Mean-proportions*, the *Extraction* of Roots by Inſpection onely, without aid of pen or compaſſes, and the like: For further diſcovery of all which, I refer you to the book it ſelf, hoping that my real intention to advance the publick good, will procure from the Ingenuous *Reader* a favorable conſtruction of what he ſhall therein find not wilfully miſtaken.

Grays-Inne
Jan. 20.
$164\frac{3}{4}$

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INſtrumental Work, as it hath the convenience of being expeditious, ſo hath it the inconvenience alſo, of being ſomwhat leſs exact, then that by Tables; yet is it exact enough for ordinary uſe, and may even where it is m ſt defective, ſerve in ſome meaſure to confirm the truth of Arithmetical Work: In this Second Edition therefore of this *Rule of Proportion*, we have ſet down the ſeveral examples in Numbers, to prove the truth of the work by *Inſtrument*; and hence the ingenuous Artiſt may eaſily learn how to prevent miſtakes in his Work, however wrought, whether by Tables, or by Inſtrument.

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UPon *the five Lines of the Rule of Proportion, there are ten ſeveral Scales projected,* viz. *two upon each common or middle Line, the one having the diviſions thereof ſhooting downwards, the other upwards :* So the firſt two Scales meet upon the middle or common Line *a b* ; the next two upon the Line *c d*, &c.

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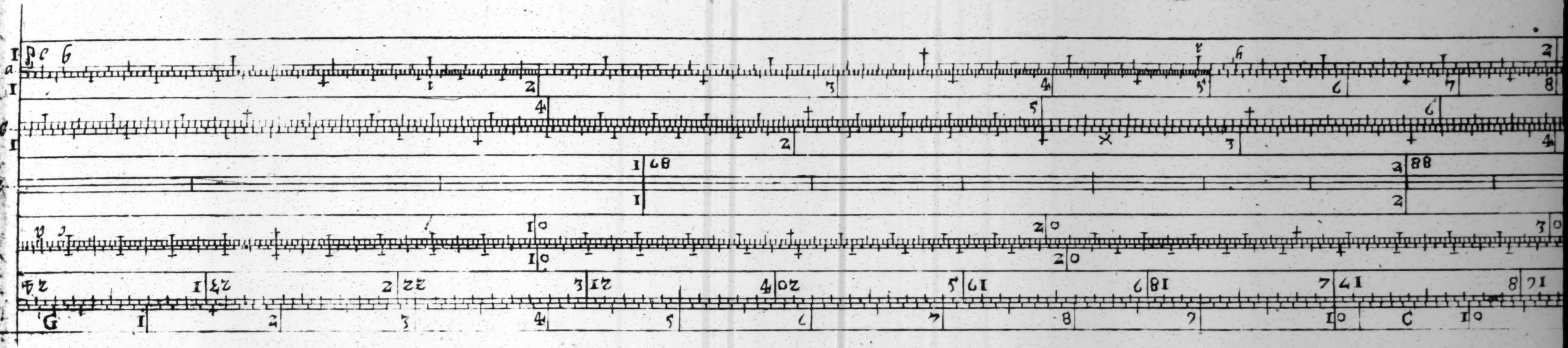

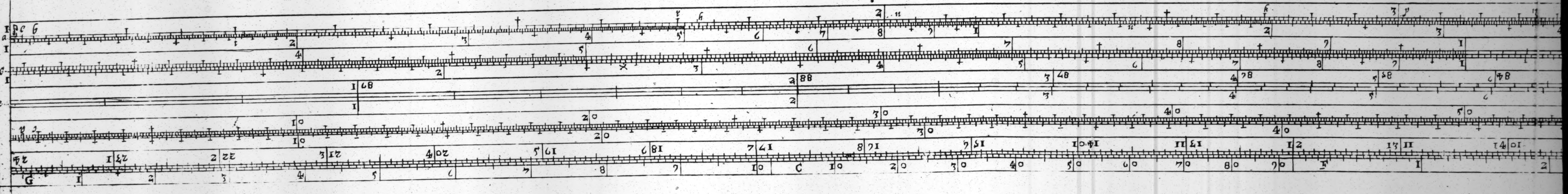

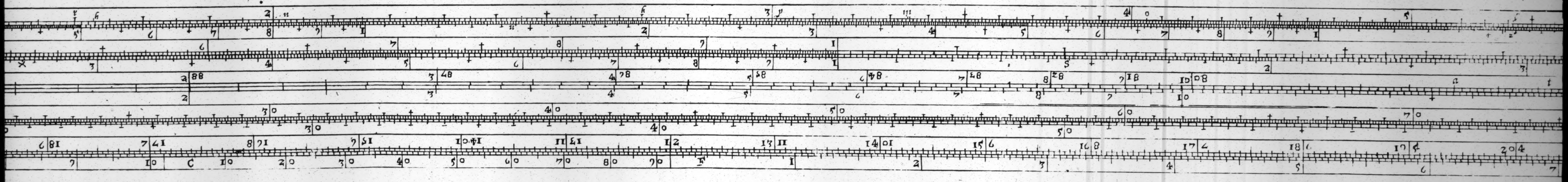

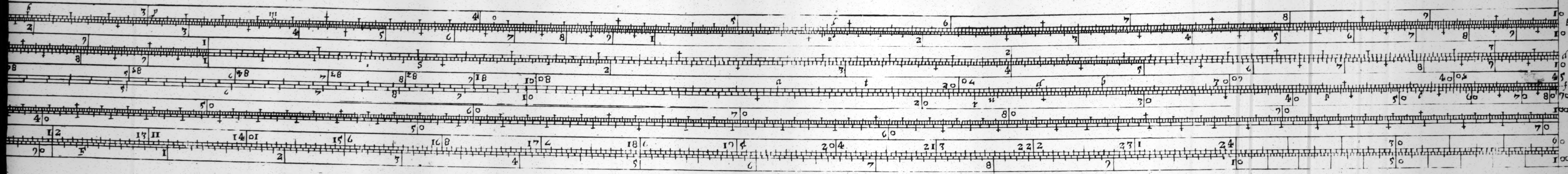

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The Use of the Rule of Proportion IN Arithmetick and Geometrie.

Cap. I.

The description of the Scales projected upon the Rule of Proportion.

UPon the five Lines of the Rule of Proportion, there are ten several Scales projected, viz. two upon each common or middle Line, the one having the divisions thereof shooting downwards, the other upwards: So the first two Scales meet upon the middle or common Line *a b*; the next two upon the Line *c d*, &c.

The uppermost, or first Scale of the Rule is a single Line of Numbers; first divided into nine unequal

unequal parts, called *Primes*, and distinguished by the figures 1. 2. 3. 4. 5. 6. 7. 8. 9. And then each of those *Primes* subdivided into other parts (according to the same Reason) called *Tenths*: And again each of those *Tenths* subdivided, or at least supposed to be subdivided into ten other parts, as the length of the Rule will admit: For *example*, upon the Scheme of our Rule (hereunto annexed) which is supposed to be about two foot and three inches long between the end-lines, in the four first *Primes* (*viz.* betwixt the figures 1 and 5) each *Tenth* is really subdivided into ten parts; but in the rest of the *Primes* (*viz.* betwixt the figure 5 and the end of that Scale) each *Tenth* is divided but into five parts; and therefore each of those five parts ought to be esteemed to have the value of 2; and the said tenth parts of those *Tenths* are hereafter called *Centesmes*: Lastly, each of those *Centesmes* is also supposed to be subdivided into ten lesser parts, which are hereafter called *Millaines*: By all which you may observe, that the longer the Rule is, the more smal divisions it will admit, and the shorter it is, the fewer.

The second Scale is another Line of Numbers thrice repeated: This Scale shoots upwards upon the common Line *a b*, and being of a lesser volume then the former, must in some parts thereof content it self with lesse divisi-

ins, *viz.* from the figure of 5 to the end of That Scale the *Tenths* are onely divided into two parts, and therefore each of those two parts ought to retain the value of five: All the three parts of this Scale being taken together, are hereafter (for distinctions sake) called the *Little Line of Numbers*, and are in their use distinguished by the first, second, and third part, as they lie in order; They are also of singular use for the ready discovery of the Cube-root, and for the resolution of other necessary operations, as shall be shewed hereafter.

The third Scale is the first Scale repeated, taking his begining from the middle of the Rule, and being broken off at the upper end thereof, is afterward continued from the lower end of the same to the place where it first began. This Scale abuts downwards upon the Common Line *c d*; and the first and this being taken together, are hereafter called the *Great Line of Numbers*, whereof the first Scale is called the first part, and this the second.

The fourth Scale is another Line of Numbers twice repeated: This Scale shoots upwards upon the Common Line *c d*, and being intirely taken together, is hereafter called the *Mean Line of Numbers:* It consisteth also of two parts, distinguished by first, and second, as they lie in order; and is of necessary use for the

the finding of the Square-root, and of me proportion, as ſhall appear hereafter.

The fift Scale is a Line of Tangents : Th Scale abuts downwards upon the Comm Line *e f*, and doth firſt contain the Artifici Tangents of the Quadrant from 0, *degr.* 35 *min.* to 45. *degr.* at the upper end of that Sca and ſo (if the Rule would permit) ſhould th be continued forward to 89 *degr.* 25 *min.* b becauſe the diviſions of that Scale being inve ted will fall out to be the ſame with the fo mer, they are to be noted and accounted bac wards from 45 *degr.* at the upper end of th Scale to 89 *degr.* 25 *min.* at the lower end the ſame; each degree thereof being ſubdivi ded into ſix parts, and each of thoſe ſix par ſuppoſed to contain ten minutes.

The ſixt Scale is a Line of Sines : Upon th Scale ſhooting upwards upon the Commo Line *e f*, are deſcribed the Artificial Sines o the Quadrant from 0 *degr.* 35 *min.* to 90 *degr* at the upper end of that Scale, each degre (upon our Rule) from 0 *degrees*, 35 *min.* to 30 *degr.* being ſubdivided into ſix parts, each part repreſenting ten minutes, as thoſe of the Tangents; but from 30 *degr.* to 50, onely into four parts, each part containing 15 minutes; From 50 to 70, into two parts, each part comprehending 30 minutes; from 70 to 85, into eaven degrees; and laſtly, from 85

degr.

degr. to 90, not divided at all, but ſuppoſed to be divided into five parts, repreſenting thoſe five laſt degrees of the Quadrant.

The ſeventh Scale ſhooting downwards, is the whole Rule divided into 1000 *equal parts*; It is hereafter called the *Scale of equal parts*, and is of uſe for the conſtruction and fabrick of the Great Line of Numbers.

The eighth Scale ſhooting upwards, is a Scale of 70 *degr.* 11 *min. of the Quadrant deſcribed according to* Mercator *and* M. Wrights *projection:* It is hereafter called the *Scale of Latitudes*, and is to be uſed together with the *Scale of equal parts*; and both of theſe taken together are uſually called the *Meridian Line*, and are of excellent uſe in Navigation, as ſhall be declared hereafter.

The ninth is the Scale of Inch-meaſure, viz. two foot thereof divided into 24 inches, and each inch into ten leſſer parts, counted both forwards & backwards, after the uſual manner.

The tenth and laſt Scale conſiſts of three ſeveral kinds, viz. a Gage-Line, a Line of Chords, and a Scale of Foot-meaſure: The firſt of theſe being ſigned by the Letter *G*, is nothing elſe but ſeven inches divided into ten equal parts, and thoſe ſubdivided into ten leſſer parts, and is hereafter to be uſed for the ready diſcovery of the *equated* diameter (and ſo by conſequent of the content) of any Wine, Beer, or Oyl-veſ-

sel: The next marked by the letter C, is an ordinary Line of *Chords*, already sufficiently known, and of frequent use amongst Artists: the third and last, marked by the letter F, is the *Scale of Foot-measure*, being nothing else but a foot first divided into ten parts, and those subdivided into ten lesser parts, and so (by consequent) the whole foot supposed to be thereby divided into 1000 parts.

At the end of these two Scales there is another double Scale placed, containing in length three inches *French*, whereof the uppermost shooting downwards, is a Scale divided into 60 parts, and that shooting upwards into 100 parts: The use of these two Scales is for the ready reduction of Sexagenarie minutes to Decimalls, and of Decimall minutes to Sexagenaries, as shall appear hereafter.

Cap. II.

The Construction and Fabrick of the Lines described upon the Rule of Proportion.

1. *To describe the* Line of Numbers, *having prepared a Rule of Silver, Brass, or Wood (of what length you please) and caused it to be ruled according to the pattern hereunto annexed, and also a Scale of* 1000 *equal parts to be drawn equal in length to your intended* Line of Numbers,

ers, repair to the Table of Logarithmes, and therein observing the first four figures of the Logarithme of 200, *besides the Index or Characteristick (viz.* 3010) *take with your Compasses the distance from the beginning of the said Scale of equal parts to the said* 3010 *parts: This done, if you apply that extent of the Compasses upwards from the begining of the* Line of Numbers, *which you intend to make, the moveable point of the Compasses will fall upon the* second Tenth *of that Line: In like manner by the first four figures of the Logarithme of* 300, *besides the Index (viz.* 4771) *you may mark the* third Tenth *of the same Line, and so consequently all the rest in their order.*

Example, If it were propounded to make a *Line of Numbers* equal to that of the first Scale, let there be a Scale of equal parts made, equal in length to that *Line*, such as the seventh Scale before described happens to be: then extending your Compasses from the begining of that Scale of equal parts to 3010, *viz.* to the point *a*, apply that extent from the beginning of your Intended *Line of Numbers*; For, that done, the moveable point of the Compasses will fall upon the *second Tenth* of that *Line*, *viz.* at the point *n*: In like manner, the extent from the beginning of the Scale of equal parts to 4771, *viz.* to the point *c* will mark out upon the intended *Line of Numbers*

ſel : The next marked by the letter C, is an ordinary Line of *Chords*, already ſufficie known, and of frequent uſe amongſt Ar the third and laſt, marked by the letter the *Scale of Foot-meaſure*, being nothing but a foot firſt divided into ten parts, thoſe ſubdivided into ten leſſer parts, an (by conſequent) the whole foot ſuppoſ be thereby divided into 1000 parts.

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1. T*O deſcribe the* Line of Numbers, *ha prepared a Rule of* Silver, *Braſs, or W (of what length you pleaſe) and cauſed it ruled according to the pattern hereunto anne and alſo a Scale of* 1000 *equal parts to be dr equal in length to your intended* Line of Nu

s, repair to the Table of Logarithmes, and rein observing the first four figures of the Lo-rithme of 200, *besides the Index or Characte-ick (viz.* 3010) *take with your Compasses the ance from the beginning of the said Scale of al parts to the said* 3010 *parts: This done, if apply that extent of the Compasses upwards m the begining of the* Line of Numbers, *which intend to make, the moveable point of the mpasses will fall upon the* second Tenth *of that e: In like manner by the first four figures of Logarithme of* 300, *besides the Index (viz.* 71) *you may mark the* third Tenth *of the ne Line, and so consequently all the rest in ir order.*

Example, If it were propounded to make a *ne of Numbers* equal to that of the first Scale, there be a Scale of equal parts made, equal length to that *Line*, such as the seventh ale before described happens to be: then tending your Compasses from the begining that Scale of equal parts to 3010, *viz.* to e point *a*, apply that extent from the begin-g of your Intended *Line of Numbers*; For, at done, the moveable point of the Compass- will fall upon the *second Tenth* of that ne, *viz.* at the point *n*: In like manner, the tent from the beginning of the Scale of e-al parts to 4771, *viz.* to the point *c* will rk out upon the intended *Line of Numbers*

the point *p*, representing the *third Tenth*
that *Line*, & so consequently the rest in ord
2. The *Line of Tangents* is framed much a
ter the same manner; for having before pr
pared a Scale of equal parts sutable to th
Line, (*viz*. consisting of half the length of t
whole Line) *Repair unto the Table of Artifi
al Sines and Tangents, and therein finding t
Artificial Tangent of* 0 *degr.* 40 *min. if (reje
ing the Characteristick or first figure thereo
you take off with your Compasses upon your fo
said sutable Scale of equal parts (as before) t
four first figures of the same Tangent (viz.* 06
*that extent being applied upwards from the b
gining of the* Line of Tangents, *will cause t
moveable point of the Compasses to fall upon the d
vision, representing* 0 *degr.* 40 *min. In like ma
ner the extent of* 1627 (*the second, third, four
and fift figures of the Tangent of* 0 *degr.* 50 *mi
will guide to mark out the same* 0 *degr.* 50 *m
upon the same Line*: And so proceeding y
may readily describe all the rest, as they f
low in order.

3. The *Line of Sines* may be drawn in t
points as the *Line of Tangents*, if you use t
second, third, fourth and fift figures of the A
tificial Sines, as you are before directed to
those of the *Tangents*. And here *note*, that t
Line before called the *Mean Line of Numb*
and these Lines of *Tangents* and *Sines* are

of them framed by one and the ſame Scale, and are alſo hereafter to be uſed together in the reſolution of *Plain Triangles*, the Scale of equal parts or *Radius*, by which they are made, being in each of them twice repeated.

4. The *Meridian Line* being framed by the ordinary Table of Meridional degrees, and the making of the *Line of chords* being obvious to every mean practitioner in the Mathematicks, I ſhall not need to trouble you with their conſtruction. The other Scales alſo, which conſiſt of equal parts, will not need any farther deſcription.

CAP. III.

Numeration upon the Rule of Proportion.

PROBL. I.

A whole number being given, to find the point where the ſame is repreſented upon the Line of Numbers.

FInd amongſt the figures, by which the Primes are diſtinguiſhed, the firſt figure of the number given; and for the ſecond figure thereof count from the beginning of the Prime, unto which the

firſt figure directs you, ſo many Tenths as that fi-gure hath Unites; Then for the third figure co[unt] from the laſt Tenth ſo many Centeſmes as the third figure hath Unites: And ſo likewiſe for the fourth figure count from the laſt Centeſme ſo ma-ny Millaines as the ſame fourth figure hath U-nites: This done, you ſhall at laſt fall upon the point where the number propounded is repreſe[nted] upon the Line of Numbers.

Example, the number given being 1728, the firſt figure thereof (*viz.* 1.) leads me unto the firſt *Prime*, deſigned by the figure 1, wi[th] which *Prime* counting ſeven *Tenths* for the ſe-cond figure, and from the ſeventh *Tenth* [two] *Centeſmes* for the third figure, and [from] the ſecond *Centeſme* eight *Millaines* [for] the fourth figure; at laſt I find the num[ber] given to be repreſented upon the firſt par[t of] the great Line of Numbers at the point *h*: [So] likewiſe is the number 27 found at the p[oint] *k*, the number 542 at the point *l*, and 334 [at] the point *m*, &c.

From hence follow theſe Corollaries:

1. *The figures which any number given [hath] towards the right hand, beſides the firſt fo[ur fi-]gures towards the left hand, are not expreſt [by] the Rule:* And therefore if the number gi[ven] were 172845, it would be likewiſe repre[ſen-]ted at the point *h*: Howbeit, that uncertai[nty] cauſeth no inconvenience in the uſe of

Rule, as shall more plainly appear hereafter.

2. *The figures by which the Primes are distinguished (in reference to one and the same number) retain always one and the same value.*

Example, in searching the number 1728, conceiving the figure prefixed at the beginning of the first *Prime* (*viz.* 1.) to have the value of *Thousands*, the figure prefixed before the second *Prime* (*viz.* 2.) ought also to be esteemed to have the value of *Thousands*, and so of the rest in their order: for according to the same reason that *h* represents 1728, the point *n* will represent 2000, the point *p* 3000, &c.

3. *The numbers, which have only the simple value of Unites, as* 1, 2, 3, 4, *&c. and those which after the first figure have nothing but Cyphers, as* 10. 100. 1000. 20. 200. 2000. *&c. are all represented at the same points.*

So 1. 10. 100. 1000. &c. may be all represented at the beginning or end of the *Line*: 2. 20. 200. 2000. &c. at the beginning of the second *Prime*: 3. 30. 300. 3000. &c. at the beginning of the third *Prime*, &c.

4. *The numbers, which being composed of three figures have a cypher in the middle, are found betwixt the beginning of the Prime, unto which they belong, and the first Tenth of the same Prime.*

So 405 begining by the figure 4 (and there-

fore to be sought for in the fourth *Prime*) represented at the point *o*.

5. *The numbers, which being composed of fo figures have two cyphers in the middle, are repr sented betwixt the beginning of the Prime, to which they belong, and the first Centesme of t same Prime.* So 1005 is found at the point q.

6. *When the Line of Numbers is repeat and for that cause consisteth of severall parts, first part thereof is in value a degree less than second, and the second a degree less than the thi &c.*

So upon the Meane Line of Numbers, if y conceive 10 at the upper end thereof to repre- sent 100, the figure 1 in the middle (or whi is all one, at the beginning of the second par will represent 10, and 1 at the lower end that Line (or which is all one, at the begin- ning of the first part) will represent 1: But 10 at the upper end thereof shall be conceived to bear but the value of 10, the figure 1 in the middle shall have the value of 1, and 1 at the lower end the value of $\frac{1}{10}$ and 2 the value of $\frac{2}{10}$ &c. In like manner, if 10 at the upper end represent 1, the figure 1 in the middle must re- present $\frac{1}{10}$, and 1 at the lower end $\frac{01}{100}$, &c.

PROBL. 2.

To finde a Fraction or broken Number upon the Line of Numbers.

THe Fractions, which are to be found upon the Line of Numbers, ought always to be Decimalls, *viz.* ought always to have for their denominators the figure 1, with nothing but cyphers towards the right hand, such as are $\frac{125}{1000}$ $\frac{25}{100}$ $\frac{5}{10}$ $\frac{75}{100}$ or the like, which may otherwise be written thus, 125, 25, 5, 75 and are equivalent to $\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$ and $\frac{3}{4}$. And therefore if the Fractions propounded be not Decimals, they ought to be reduced to such: For, that done, they may be discovered in all points as whole numbers are found out upon the Line, which may be plainly understood by the *examples* produced in the sixt *Corollary* of the last Probleme.

PROBL. 3.

To finde a Mixt Number upon the Line of Numbers.

F*Irst finde by the first Probleme aforegoing the point representing the whole parts of the number given, and then afterwards the Fraction or broken parts thereof in the ranks that follow.*

Example, a Line that hath the length of 17 foot

foot and $\frac{18}{103}$ of a foot (which may more conveniently be written thus, 17.28.) being propounded, first I finde the whole part thereof (*viz*. 17) represented at the point and after counting two Centesmes, and the eight Millaines, at last I find the number given to be represented at the point *h*: In like manner if the number propounded were 172. 8, or 1.728, it would be still represented at the same point.

Probl. 4.

Any point of the Line of Numbers being assigned, to find the figures represented at the same point.

Take the figure prefixed at the beginning of the Prime, within which the point is propounded, for the first of the figures required; then shall the second figure required be composed of so many Unites as there are Tenths intercepted betwixt the beginning of the same Prime and the point given : In like manner shall the third figure required have so many Unites as there are Centesmes comprehended betwixt the last of those Tenths and the said point : And so likewise shall the fourth figure consist of so many Unites as there are Millaines between the last Centesme and the point given.

Example, If the point *h* were propounded, because that point is situate within the Prime,

be-

before which the figure 1 is prefixed, I take the figure 1 for the first of those required; And then finding seven Tenths betwixt the beginning of that Prime and the point given, I set down 7 for the second: And so proceeding and finding two Centesmes betwixt the last Tenth and the said point, I take 2 for the third figure: And lastly conceiving eight Millaines to be comprehended between the last Centesme and the point given, I take 8 for the fourth figure required: This done, I conclude, that the figures represented at the point propounded, are 1728. In like manner the point *q* being given, I take 1 for the first figure; but here because I find no Tenths betwixt the beginning of that Prime and the point given, I write a cypher in the second place; and there also finding no Centesmes I write also a cypher in the third place; And then at last finding the point propounded in the middle of a Centesme (which is supposed to be divided into ten Millaines) I annex in the fourth place 5: This done, the figures represented at the point given will be found 1005.

Probl. 5.

An Arke or Angle being propounded to find upon the Rule of proportion the point which represents the Tangent of the same Arke or Angle.

If the Arke or the measure of the Angle exceed not 45 *degrees, search the degrees of that Arke or Angle upon the Line of Tangents, mounting upwards from the lower end of that Line towards the upper end of the same.*

So the Tangent of an Arke or Angle, which consists of 15 degrees, is represented at the point *a*: of 25 degrees at the point *b*, &c.

But if the Arke or measure of the Angle exceeds 45. *degrees, look the degrees thereof, descending downwards from the upper end of the Line towards the lower end of the same.* So the Tangent of 65 degrees is found at the point *b*, of 75 degrees at the point *a*, &c.

And if the Arke or Angle propounded (besides the whole degrees) is also composed of certain minutes, find first the whole degrees, and after that, betwixt the last degree found and the next that follows, take so many of the parts which may amount to the minutes given, accounting each of the parts contained betwixt the two degrees for ten minutes: So the Tangent of 22 *degr.* 45 *min.* is found at the point *d*, and the Tangent of 72 *degr.* 45 *minutes* at the point *t*. And therefore

e con-

è converso, if the points *d* and *t* were given upon this Line, the degrees and minutes represented by them would be 22 *degr.* 45 *min.* and 72 *degrees* 45 *min.* &c.

PROBL. 6.

An Ark or Angle being propounded, to find upon the Rule of Proportion the point, which represents the Sine of the same Ark or Angle.

Find upon the Line of Sines the degrees of the Ark or Angle given, and you have your desire: So the Sine of the Ark or Angle of 22 *degr.* is represented at the point *r*.

But if the Ark or Angle given have also minutes annexed, first search the whole degrees given, and then between that degree found and the next that follows, take so many parts as you have minutes propounded, conceiving the distance betwixt each degree and the next that follows to comprehend 60 minutes.

So the Sine of 22 *degr.* 45 *min.* is found at the point *u*; of 42 *degr.* 50 *min.* at the point *q*; of 52 *degr.* 45 *min.* at the point *c*, &c. And therefore here also *è converso*, if the points *u*, *q*, and *c* were assigned upon this Line, the degrees and minutes represented by them would

be

be 22 *degr.* 45 *min.* 42 *degr.* 50 *min.* and 52 *degr.* 45 *min.* &c.

CAP. IV.

The use of the Rule of Proportion in Arithmetick.

IN *Arithmetick* there are three several sorts of Proportion, *Arithmetical*, *Geometrical*, and *Musical*. *Arithmetical*, when diverse numbers being compared together retain amongst themselves equal differences, as these, 2. 4. 6. 8. &c. And this is either *continued*, as in the numbers before produced, or in these 3. 6. 9. 12. 15. &c. which is also called *Arithmetical* Progression, or a rank of numbers *arithmetically* proportional: or discontinued, as in these, 2. 4. 10. 12. or the like. *Geometrical* proportion is, when diverse numbers being compared together differ amongst themselves according to the same rate or reason, as these, 2. 4. 8. 16 &c. For here, as 2 is half 4, so is 4 half 8, and 8 half 16: this is likewise either *continued*, as in those before propounded, or in these, 1. 3. 9. 27. 81. &c. or the like, which is also called *Geometrical* progression, or a rank of numbers *geometrically* proportional: Or *discontinued*, as in these, 2. 4. 16. 32: for as 4 is

4 is double 2, ſo is 32 double 16, but ſo is not 16 being compared with 4. *Muſical* proportion is that which doth as it were proceed from both the former, as when three numbers or terms being propounded, the firſt bears the ſame proportion to the third, that the difference betwixt the firſt and the ſecond bears to the difference betwixt the ſecond and third. as in theſe, 3. 4. 6. for here, as 3 is half 6, ſo is 1, the difference betwixt 3 and 4, to 2, the difference betwixt 4 and 6. So likewiſe 2. 3. 6. and 10. 16. 40 are ſaid to be numbers *muſically* proportional: For in the firſt of theſe two laſt examples, as 2 is to 6, ſo is 1 to 3; And in the other as 10 is to 40, ſo is 6 to 24. Thus have I here thought fit briefly to remember the *Reader* of the ſeveral kinds of Proportion, which he doth uſually find in the writings of thoſe that treat of *Arithmetick*; to the end that the *Problemes* which follow both in *Arithmetick* and *Geometry* may be the better underſtood.

Probl. I.

Two Numbers being given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

TO perform this Probleme Arithmetically, you muſt multiply the ſecond number given

given by the firſt, the Product is the third pr portional ſought, which being multiplied b the firſt continually, will conſtitute the four and fift proportional, &c. And becauſe th Multiplication by Logarithmes is perform by Addition, if to the Logarithme of the ſeco number given you add the Logarithme of t firſt, the abſolute number anſwering to th ſumme, ſhall be the third proportional; a the Logarithme of the firſt being added to th ſum, ſhall give the Logarithme of the four Proportional, and ſo forward.

Example, Let it be propounded to find third Proportional to theſe two numbers and 4.

The Logarith. of 2 *is*	0. 301030.
The Logarith. of 4 *is*	0. 602060.
Their ſumme	0. 903090.

is the Logarithme of 8 *the third Proportional.*

And to perform this by our Rule of Pro portion, *Extend the Compaſſes upon the Line Numbers from one of the numbers given to t other; this done, if you apply the ſame ext (upwards or downwards) from either of the num bers propounded, the moveable point of the Com paſſes will fall upon the third proportional requir And ſo the ſame extent being applied the ſam way from the third, the moveable point of t Compaſſes will fall upon the fourth Proportion and from the fourth upon the fifth, &c.*

Exam

Example, Let it be propounded to finde a third proportional to these two numbers 2 and 4, which may bear the same proportion to 4, that 4 bears to 2; First, I extend the Compasses upon the first part of the Mean Line of Numbers from 2 to 4; this done, if I apply that extent out-right from 4 upwards, the moveable point of the Compasses will fall upon 8 the third proportional required; and being applied the same way from 8, the moveable point will rest upon 16, the fourth proportional; and from 16 to 32, the fifth; and from 32 to 64, the sixth proportional. But now if you would yet continue the Progression farther, and so finde the next proportional to 64 (because the movable point in that case will fall beyond the Line) apply that extent the same way from 64 in the first part of that Line; which done, the movable point of the Compasses will then fall upon 128 the seventh proportional; and so proceeding farther you may find 256, the eighth; 512, the ninth, &c.

Contrariwise, if it were required to find a third proportional to the same numbers 2 and 4, which may bear the same proportion to 2, that 2 bears to 4; extend the Compasses upon the second part of the Mean Line of Numbers from 4 to 2 downwards; this done, if you apply that extent from 2 the same way (*viz.*

downwards) the movable point will fall upon 1, the third proportional required ; And from 1 upon $\frac{5}{10}$ or .5, by the last Corollary of the third Chapt. and from .5 to .25, by the same Corollary, &c.

In like manner, if the two numbers given were 10 and 9, the Compasses being extended downwards from 10 at the upper end of the same Line of Numbers to 9, and that extent applied from 9 the same way, the moveable point of the Compasses will rest upon 8.1, the third proportional (for the given numbers being 10 and 9, common sense tells me that it cannot be 81, & therfore ought to be 8.1) and from 8.1 the movable point will fall upon 7.29, the fourth proportional, &c. So likewise if the numbers propounded were 1 and 9, conceiving 10 at the upper end of the Line to represent 1, extend the Compasses from thence to 9, which extent being applied downwards from 9, will cause the movable point of the Compasses to fall upon 81, the third proportional; and from 81 upon 729, the fourth proportional, &c. And therefore *note* hence that 1 at the beginning, 1 in the middle, and 10 at the end of the Line, are all arbitrary points, and may each of them represent sometimes 1, sometimes 10, somtimes 100, sometimes 1000, &c. as the terms by which you are to work, shall require according to the

third

third Corollary of the third Chapter.

Neverthelefs neither do the *examples* before produced, nor thofe which fhall follow in the infuing Problemes, at all croffe that which hath been formerly taught in the fecond Corollary of the third Chapter: For in the laft *example*, the end of the Line in regard of the firft term given (*viz.* 1) hath the fingle value of an Unit; but in refpect of the fecond term 9, it challengeth the value of 10; and in reference to the third number 81, the value of 100, &c.

Laftly, if the the numbers given were 10 and 12, the third proportional upwards would be 14.4, the fourth 17.28, &c. and the numbers 1 and 12 being propounded, the third proportional upwards (as before) will be 144; the fourth 1728, &c.

The like operations may be alfo performed (and that much more exactly) upon the great Line of Numbers: For *example*, 1 and 4 being given, I defire to know a third, a fourth, a fifth, &c. geometrically proportional: To perform this, extend the Compaffes upon that Line a crofs from 1 at the begining of the fecond part thereof unto 4 upon the firft part of the fame; which done, that extent being applied the fame way, (*viz.* upwards and acroffe) will reach from 4 upon the

first part unto 16 upon the second, and from thence to 64 upon the first part again, &c.

PROBL. 2.

One number being given to be multiplied by another number given, to find the Product.

TO resolve this Probleme Arithmetically whether by natural or by artificial numbers, the Proportion is;

As 1 is to the Multiplicand, so is the Multiplicator to the Product.

Example, Let 30 be the Multiplicand, and 25 the Multiplicator.

I say, as	1	0.00000
Is to	30	1.47712
So is	25	1.39794
To	750	2.87506

To perform this by our Rule of Proportion, *Extend the Compasses upon the Line of Numbers from 1 unto the Multiplicator; This done, if you apply that extent the same way from the Multiplicand, the movable point of the Compasses will fall upon the Product required.*

1. *Example*, Let the Multiplicator given be 25, and the Multiplicand 30: Here if you extend the Compasses upon any of the Lines of Number from 1 unto 25, and then apply that extent the same way from 30, the movable

vable point of the Compaſſes will fall upon 750, the product required. So 1.728, and 25. 6 being propounded to be multiplied, the product will be found 44.2.

2. *Example*, The two numbers given being 45 and 25, I extend the Compaſſes upon the ſecond part of the Mean Line of Numbers from 1 to 25; Then (becauſe, if *I* apply that extent the ſame way from 45 upon the ſame part of that Line, the movable point will fall beyond the Line) I apply the ſame extent the ſame way from 45 in the firſt part thereof; which done, the movable point will fall upon 1125, the product deſired: So the two numbers given being 1.728, and 64. 5, the product required will be 111.4.

3. *Example*, If 75 and 35 were given to be multiplied, the Compaſſes ought to be extended downward from 1 to 75 in the firſt part of the Mean Line of Numbers, or (which is all one) from 10 at the upper end of that Line to 75; for, that extent being applied the ſame way from 35, will cauſe the movable point of the Compaſſes to fall upon 2625 the product required.

4. *Example*, If it were required to find the content of a piece of ground 8. 75 perches long, and 6. 45 broad; becauſe this queſtion is reſolved by multiplying the length by the breadth, I extend the Compaſſes from 10 at

the top of the Line to 8. 75; then applyin[g] that extent the ſame way from 6. 45, the mo-vable point will fall upon 56. 4, the conten[t] required, *viz.* 56 Perches and $\frac{4}{10}$ or .4 of [a] Perch.

And here you may obſerve, that theſe la[st] *examples*, and thoſe that are like unto the[m] may likewiſe be performed in working up-wards; But in ſuch caſes to ſhun too great a[n] extent of the Compaſſes, it is better to beg[in] the operation from 10 at the top of the Lin[e] and ſo to deſcend downwards according t[o] the inſtructions before delivered: For (tak[e] this for a general Rule once for all, that) *A[ll] operations, which are wrought upon the Rul[e of] Proportion, are beſt performed, when the leg[s of] the Compaſſes have the leaſt extenſion.*

Again, becauſe this Probleme of *Multipl[i]cation*, as alſo (for the moſt part) all the r[est] that follow are reſolved by the finding out [of] a fourth number *geometrically* proportional t[o] three other numbers given, we will therefo[re] here inſert this other advertiſement: Whe[n]ſoever queſtion is made of finding a fourt[h] proportional to three ſuch numbers given, f[or] the better conveniency of working upon th[e] Rule the order of the ſecond and third term[s] may be changed, ſo that always care be taken that the firſt number may ſtill retain the fir[st] place: For *example*, you may ſay, as 1 is t[o]

25

25, so is 30 to 750; or as 1 is to 30, so is 25 to 750. And this Rule is diligently to be observed in Multiplication, Division, the Rule of three direct, the resolution of Plane and Spherical Triangles, and generally in all questions of such like proportions; to the end that in working upon the *Rule of Proportion* we may alwaies avoid too great an extension of the Compasses, and by that means perform the work more exactly.

Lastly, here observe, that Multiplication, and all other questions hereafter produced, which may be wrought upon the Mean Line of Numbers, may likewise be performed upon the Great Line of Numbers (and that much more exactly) by working either outright or acrosse, as the questions propounded shall require; which (I well hope) I may hereafter leav to the discretion of the ingenious Reader to discover, without any further instruction, they being (indeed) but one and the same *Instrument* represented in differing postures.

PROBL. 3.

A number being propounded to be divided by another number, to find the Quotient.

THe resolution of this Probleme dependeth upon this analogie.

As the Divisor is to the Dividend, so is 1 to the Quotient.

Example, Let 750 be the Dividend, and let the Divisor be 25.

I say as	750	2. 87506
Is to	25	1. 39794
So is	1	0. 00000
To	30	1. 47712

To perform this by our Rule of Proportion, *Extend the Compasses upon the Line of Numbers from the Divisor to 1; This done, if you apply that extent the same way from the Dividend, the movable point will fall upon the number of the Quotient.*

1. *Example*, Let 750 be the number given to be divided by 25, the divisor: I extend the Compasses downwards from 25 to 1; then applying that extent the same way from 750, at last the movable point will fall upon 30, the Quotient required.

2. The number 1125 being given to be divided by 25; I extend the Compasses downwards from 25 to 1, then applying that extent the same way from 1125, the movable point will fall upon 45, the Quotient required. The same Quotient will also be found, if changing the terms you first extend the Compasses from 25 to 1125, and then apply that extent from 1; for so also shall the movable point fall upon 45, as before; according-

ing

ing to the obſervation made in the laſt Probleme: In like manner 111.4 being propounded to be divided by 1.728, the quotient will be found 64.5.

3. The number 2625 being propounded to be divided by 75; extend the Compaſſes upwards from 75 in the firſt part of the Mean Line of Numbers to 1, or (which is all one) from 75 in the ſecond part to 10 at the top of the Line; This done, if you apply that extent the ſame way from 2625, the movable point wil from thence reach to 35, the quotient required: So likewiſe 56.4 being given to be divided by 8.75, the Quotient will be 6.45.

Now to diſcover of how many figures any Quotient ought to conſiſt, it will be neceſſary to obſerve how many times the Diviſor may be written under the Dividend according to the rules of Diviſion; for, of ſo many figures ſhall the Quotient be compoſed: for *example*, 12231 being given to be divided by 27; becauſe the Diviſor 27 may (according to the Rules of Diviſion) be written three times under the Dividend 12231 (as may appear by this *example*) I ſay, that the Quotient, which is produced by the diviſion of 12231 by 27 conſiſts of three figures: For, having extended the Compaſſes downwards in the ſecond part of the Mean Line of Numbers from

12231
27.

27 (the Divisor) to 12231 (the Dividend) and applied that extent the same way from 1, the movable point will fall in the first part upon 453, the Quotient of 12231 divided by 27.

PROBL. 4.

To three numbers given to find a fourth in a direct proportion.

TO resolve this Probleme the analogy is; As the first number given is to the second, so is the third number to the fourth.

Example, let the three numbers given be 7. 14. 22.

I say as	7	0.84509
Is to	14	1.14613
So is	22	1.34242
To	44	1.64346

To perform this by the Rule, *Extend the Compasses from the first number or term given unto the second; which done, that extent being applied the same way from the third term, will cause the movable point to fall upon the fourth term required.*

Example, if the circumference of a Circle, whose Diameter is 7, be 22; what circumference will a Circle have, whose Diameter is 14? Extend the Compasses upwards upon the Mean Line of Numbers from 7 in the first part thereof, unto 14 in the second; This done,

done, that extent being applyed the ſame way from 22, will make the movable point reſt upon 44, the circumference required.

Or otherwiſe downwards; The circumference of a Circle being 22, and the Diameter thereof 7, how much ſhall the Diameter of a Circle be, whoſe circumference is 44? Extend the Compaſſes downwards from 22 in the ſecond part, to 7 in the firſt; which done, that extent being applied the ſame way from 44, will reach to 14, the Diameter ſought for.

PROBL. 5.

To three numbers given to find a fourth in an inverſed proportion.

TO reſolve this Probleme the analogy is As the third number is to the ſecond, ſo is the 1 to the fourth.

Example. If 60 men do make a trench in 45 hours, in what time will 40 men make ſuch another.

I ſay, as	40	1.69206
Is to	45	1.65321
So is	60	1.77815
To	67.5	1.82930

To perform this by our Rule of Proportion, *Extend the Compaſſes upon the Line of Numbers from the firſt of the numbers given to the ſecond, having both the ſame denomination; this done,*

done, if that extent be applied quite backwar[ds] from the third given number, the movable poi[nt] will fall upon the fourth number you look for.

Example, if 60 Pioners can make a tren[ch] of a certain length and breadth in 45 hour[s] how long will it before 40 men can ma[ke] such another? Extend the Compasses fro[m] 60 to 40 (those terms having both the sam[e] denomination, *viz.* of men) This done, tha[t] extent being applied backwards from 45, wi[ll] reach to 67.5, the fourth number you loo[k] for; I conclude therefore that 40 men wi[ll] perform as much in 67 hours and an half, a[s] 60 men will do in 45 hours.

PROBL. 6.

To three numbers given to find a fourth i[n] a doubled proportion.

THe use of this Probleme appears chiefly i[n] proportions of Lines to superficies, or o[f] superficies to Lines.

Now if the denomination of the first and second terms be of Lines, *extend the Compasse[s] upon the Line of Numbers from the first term to the second; this done, that extent being applied twice the same way from the third term, wil[l] cause the movable point to fall upon the fourth term required.*

Example, if the content of a Circle whose

Diameter

Diameter is 14 inches, be 154, what will the content of a Circle be, whoſe Diameter is 28? Here 14 and 28 having the ſame denomination (*viz.* of Lines) I extend the Compaſſes from 14 to 28; then applying that extent the ſame way from 154, the movable point will firſt fall upon 308, and from thence upon 616, the content deſired.

But if the firſt two terms have the denomination of *areas* or contents, and the *quæſitum* be a Line, this is the Rule: *Extend the Compaſſes upon the Mean Line of Numbers from the firſt term to the ſecond; this done, that extent being applied the ſame way upon the Great Line of Numbers from the third term, will cauſe the movable point to fall upon the fourth term required.*

Example, If the Diameter of a Circle, whoſe *area* is 154, be 14; what Diameter will a Circle have, whoſe *area* is 616? Extend the Compaſſes upon the Mean Line of Numbers from 154 to 616; which done, that extent being applied the ſame way upon the Great Line of Numbers from 14 will reach to 28, the Diameter required.

PROBL. 7.

To three numbers given to find a fourth in a tripled proportion.

THe uſe of this Probleme appears in the proportion of Lines to Solids, & *contrà*.

If

If therefore the first and second terms ha[ve] the denomination of Lines, *extend the Com[passes] upon the Line of Numbers from the fir[st] term to the second; this done, and that ext[ent] applied three times the same way from the thi[rd] term, will cause the movable point at last to fa[ll] upon the fourth term required.*

If an Iron Bullet, whose Diameter is 4 in[ches], weigheth 9 pounds, what is the weig[ht] of another Iron Bullet, whose Diameter is [8] inches? Extend the Compasses from 4 to 8, which done, and that extent applied the sam[e] way three times from 9, the movable poi[nt] will first fall upon 18, then from 18 upon 3[6], & at last from 36 upon 72, the weight require[d].

But if the first two terms be weights or con[tents] of Solids, and a Line is sought for: *ex[tend] the Compasses upon the Little Line of Num[bers] from the first term to the second; This don[e], and that extent applyed the same way upon t[he] Great Line of Numbers from the third term, w[ill] cause the movable point of the Compasses to fa[ll] upon the fourth term required.*

If the side of a Cube weighing 72 pound[s] be 8 inches, how many inches is the side of [a] Cube that weighs 9 pounds? Extend th[e] Compasses downwards upon the Little Lin[e] of Numbers from 72 to 9; that done, and th[e] same extent applyed the same way upon th[e] Great Line of Numbers from 8, will cause the

movable

movable point to fall upon 4, the side required.

PROBL. 8.

Betwixt two numbers given to find a mean arithmetically proportionall.

THis Probleme may be performed without the help of the *Rule of Proportion* : Neverthelesse, because it conduceth to the resolution of the next insuing Probleme, I insert it in this place, and give this Rule for it :

Add half the difference of the given terms to the lesser of them : for that aggregate is the arithmetical mean required.

Example, Let 10 and 40 be the terms given: here, if you substract the one out of the other, their difference will be found 30 , whose half (15) being added to 10, the lesser term, their summe (25) is the Arithmetical mean you look for.

PROBL. 9.

Betwixt two numbers given, to finde a mean musically proportional.

BOëtius (*lib.* 2. *Arith. cap.* 38.) hath this Rule for it : *Differentiam terminorum in minorem terminum Multiplica , & post, junge terminos , & juxta eum, qui inde confectus est, committe illum numerum , qui ex differentiis & termino minore productus est , cujus cum latitudinem inveneris , addas eam minori termino, & quod*

quod inde colligitur medium terminum pon

Multiply the difference of the terms by the lesser term, and add likewise the same terms together: this done, if you divide that product by the summe of the terms, and to the quotient thereof adde the lesser term, that last summe is the musicall mean desired.

Or shorter thus:

Divide the product of the given terms by their summe: for, this done, the quotient doubled is the mean required. So the Numbers given being 6 and 12, I say 12 multiplyed by 6 make 72, which divided by 18 (the summe of 12 and 6) leavs 4 in the quotient, whose double (8) is the musicall mean you look for. This Probleme therefore may be performed by the second and third aforegoing: or yet otherwise thus:

Find the arithmeticall mean betwixt the numbers given, and then the analogie will be this.

As the arithmeticall mean found
Is to the greater extreme:
So is the lesser extreme
To the musical mean required.

Example, 10 and 40 being propounded, the arithmeticall mean betwixt them (by the last Probleme) is 25: I say then, As 25 is to 40, so is 10 to 16, the musicall mean desired: the term therefore here sought for may be discovered by the fourth Probleme aforegoing.

And

And here (I conceive) it will not be amisse to observe, that by this last Rule, having any two numbers propounded, you may interject two other numbers betwixt them, in such sort that they four being in severall relations compared one with another, may contain in them all the three proportions above-mentioned, which kind of Harmony *Boetius* (*lib.* 2. *cap. ult.*) calls *Maxima & perfecta symphonia*: So in the numbers before-mentioned 10, 16, 25, and 40, if you compare 10, 25, and 40 together, there shall you find *Arithmetical* proportion; 10, 16, and 40 together, there Harmony, or *Musicall* proportion; if all of them together, there have you *Geometricall* proportion discontinued: For as 10 to 16, so 25, to 40, And this is that *Harmony* which the same *Boetius* (in the same place) affirmeth to have *Magnam vim in Musici modulaminis temperamentis, & in speculatione naturalium quæstionum*: Great force in the composure of Musick, and in the discovery of the secrets of Nature: And therefore he also averreth in another place (*viz. lib.* 1. *cap.* 2.) that *the reason of Numbers was the chiefest Rule, according to which Almighty God framed the world*: According to that testified of the Wisedome of God (in the Wisedome of *Sal.* cap. 11 *v.* 20. *Thou hast ordered all things in measure, and number, and weight.* The Statists also and Politicians fetch much

from these three proportions for the regular direction of a wel governed Common-wealth as may be easily collected out of their writings and is learnedly proved by *Bodin* in the last chapter of his Common-wealth.

PROBL. 10.

Betwixt two numbers given to find a mean geometrically proportionall.

Extend the Compasses upon the mean Line of Numbers from one of the numbers given to the other; this done, and the same extent applyed upon the Great Line of Numbers from either of those numbers towards the other, the movable point will fall in the middle betwixt them, viz. *upon the point representing the mean proportionall required.*

Example, 8 and 32 being propounded, the mean proportionall between them will be found 16: For if I extend the Compasses upon the Mean Line of Numbers, from 8 in the first part thereof to 32 in the second, and afterwards apply that extent upon the Great Line of Numbers from 8 towards 32, the movable point will fall upon 16, the mean proportionall demanded; for as 8 is to 16, so is 16 to 32: so the mean betwixt 6.4, and 14.4, is 9.6, &c.

PROBL.

PROBL. 11.

Betwixt two numbers given, to find two means geometrically proportional.

EXtend the Compasses upon the Little Line of Numbers from one of the numbers given to the other: this done, and that extent applied upon the Great Line of Numbers from either of those numbers towards the other, will cause the movable point to fall first on the third part of the distance between them, viz. *upon the point representing one of the mean numbers required, and being applied again the same way, will at last rest upon the other proportionall you look for.*

Example, Let 8 and 27 be the two numbers between which two mean proportionalls are desired: first, I extend the Compasses upon the Little Line of Numbers upwards from 8 to 27: then applying that extent twice upon the Great Line of Numbers from 8 towards 27, I find the movable point to fall first upon 12, and then upon 18, which are the two means you desire to know: for as 8 is to 12, so is 12 to 18, and 18 to 27.

PROBL. 12.

To find the Square-root of any number under 1.000.000.

THe *Extraction* of Roots, which is accounted the hardest Lesson in *Arithmetick*, is

performed by the help of this *Inſtrument* wit greateſt eaſe and dexterity : for, whereas th *Problemes* before premiſed, as alſo thoſe tha follow, cannot well be expedited without th joint uſe of the *Rule* and *Compaſſes* together theſe of the *Extraction* of the *Square* and *Cub* Roots may be reſolved onely by *Inſpection* without any trouble at all, or aid of Compaſ ſes: ſo that a man either riding or going in ha may immediately read upon the *Rule* the roo of any *Square* or *Cube* number propounded which compendious way of *Extraction* can not chooſe but prove to be of admirable uſ eſpecially in queſtions that concern Militar Orders, as ſhall more plainly appear hereaf ter. Wherefore to extract the *Square-root* pro ceed thus :

1. *When the figures of the number given a even,* viz. *when the number conſiſts of two, fou or ſix figures, look the ſame number in the fir part of the Mean Line of Numbers : which don juſt at the ſame point ſhal you likewiſe find upon th Great Line of Numbers the Square root you look fo*

Example, 264.196 being propounded, th Square-root thereof will be found 514 : for find the number 264.196 repreſented in th firſt part of the Mean Line of Numbers at the point *x*, and at the ſame point upon the ſe cond part of the Great Line of Numbers I ob ſerve 514, the Square-root required.

2. *When*

2. *When the figures of the number given are odde, viz. one, three, or five, ſearch the ſame number in the ſecond part of the Mean Line of Numbers: which done, juſt at the ſame point upon the Great Line of Numbers ſhall you find alſo the Square-root demanded.*

Example, 144 being propounded, I demand the Square-root thereof: that number I find to be repreſented in the ſecond part of the Mean Line of Numbers at the point *s*, and juſt there alſo upon the Great Line of Numbers I diſcover 12, which is the Square-root of the number propounded. So likewiſe is 144 the Square-root of 20.736.

PROBL. 13.

To extract the Cube-root of any number under 1.000.000.000.

1. WHen the number propounded conſiſts of *one, four, or ſeven figures, find it in the firſt part of the Little Line of Numbers: that done, at the ſame point upon the firſt part of the Great Line of Numbers, you ſhall find the Cube-root you look for.*

Example, Let the number given be 1728 whereof the Cube-root is required: I find that number in the firſt part of the Little Line of Numbers at the point *t*, and at the ſame point upon the Great Line of Numbers I alſo diſco-

ver 12, the Cube-root desired: In like manner is 12.50 the Cube-root of 1950, and 144 the Cube-root of 2.985.984.

2. *When the number given consists of two, five, or eight figures, search it in the second part of the Little Line of Numbers, and then proceeding as before, you shall have your desire.*

Example, if 14.348.907 were given, the Root thereof would be found 243: for, that number being found in the second part of the Little Line of Numbers at the point *u*, just at the same point upon the Great Line I also find 243, the Cube-root required.

3 *When the number propounded consists of three, six, or nine figures, look for it in the third part of the Little Line of Numbers: for so likewise at the same point upon the Great Line will appear the Root required.*

So the number 159.220.088 being found in the first part of the Little Line of Numbers at the point *z*, his Cube-root is there likewise found upon the great Line of Numbers to be 542: And the Cube-root of 159.220 is found to be 54.2, &c.

The order of finding out the Cube-numbers upon the several parts of the Line may be fitly expressed by this Figure.

1	2	3
1	2	3
4	5	6
7	8	9

CAP.V.

CAP. V.

The use of the Rule of Proportion in Geometrie, viz.

In the Dimension.

1. Of Plane Triangles.

PROBL. I.

The three Angles and one side being known, to find the other two sides.

TO resolve this Probleme, this is the *Analogie*: As the Sine of the Angle opposed to the side known, is to the parts of the same side: so is the Angle opposed to one of the sides unknown, to the parts which measure that side.

Illustration by Numbers.

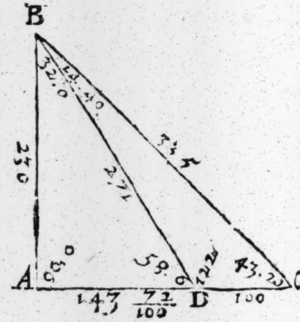
As Sine	C B D.	14. 40	9. 40349
To Side	D C.	100	2, 00000
So Sine	A D B.	58	9. 92842
To Side	B C.	335	2. 52493

And therefore

Extend the Compasses acrosse from the Sine of the Angle opposed to the Side known, to the same Side, found upon the mean Line of Numbers: then applying

plying that extent the same way from the Sine of the Angle opposed to one of the Sides required, the movable point will fall upon the parts which measure that required Side.

Example, in the Triangle C B D, let the Angle C be 43 *degr.* 20 *min.* the Angle D 122 *degr.* and by consequent the Angle B (being the complement of the two other Angles to 180 *d.* or two right Angles) 14 *deg.* 40 *min.* and let the Side D C, being 100 pace, represent the distance between the two stations D & C: I demand then the distance between C & B. Extend the Compasses acrosse from 14 *d.* 40 *m.* upon the Line of Sines to the middle of the Mean Line of Numbers representing 100; then that extent being applied the same way from 122 *d.* upon the Line of Sines, or (which is all one) from 58 *degr.* (for by the Rules of *Trigonometrie* the Sine of an obtuse Angle and that of his complement to 180 is one and the same Line) will cause the movable point to fall upon 335, and so many paces is the distance required: In like manner, the extent being applied the same way



from

from 43 *degr.* 20 *min.* upon the Line of Sines, the movable point will fall upon 271, the parts of the side D B.

Or otherwise, by changing the terms of the *Analogie*, thus:

Extend the Compasses outright upon the Line of Sines from 14 *d.* 40 *m.* to 58 *d.* then applying that extent the same way upon the Line of Numbers from 100, the movable point will rest upon 335, the distance required: So likewise the Compasses being extended outright upon the Line of Sines from 14 *d.* 40 *m.* to 43 *d.* 20 *m.* and that extent applied the same way upon the Line of Numbers from 100, the movable point wil fal upon 271, the parts of the Side D B.

And here observe, that not onely this present Probleme, but also all those that follow (which concern the resolution of Triangles) may be resolved two manner of ways, *viz.* by working either out-right, or acrosse, except some few, which we intend to mark in their proper places. Remember likewise what hath been before touched in the second Chapter aforegoing, *viz.* that the Mean Line of Numbers is the onely *Line* to be used with these of Sines and Tangents, and no other.

PROBL.

PROBL. 2.

By the knowledge of two sides and an Angle opposed to one of them, to find the other two Angles and the third side.

THis is the *Inverse* of the last Probleme: for, as the side opposed to the given Angle, is to the Sine of the same Angle: so is the other side known, to the Sine of the Angle thereunto opposed;

Illustration by numbers.

As Side BC. 335.	2.52493
To Sine ADB. 58.	9.92842
So Side DC. 100.	2.00000
To Sine CBD. 14.40.	9.40349

And therefore

Extend the Compasses acrosse from the parts of the side opposed to the Angle known, unto the Sine of the same Angle: then that extent being applied the same way from the parts of the other known side, will cause the movable point to fall upon the Sine of the Angle required.

So in the foresaid Triangle *C*, *B*, *D*, the side *C B*, being 335, the Angle *D* (opposed thereunto) 122 *d.* 0 *m.* and the side *D C*, 100, the Angle *B*, will be found 14 *d.* 40 *m.* For if you extend the Compasses acrosse from 335 upon the Line of numbers, to 122 *d.* 0 *m.* (or rather to 58 *d.* 0 *m.* as aforesaid) upon the Line of

of Sines, and after apply that extent the ſame way from 100 upon the Line of Numbers, the movable point will reſt upon 14 *d.* 40 *m.* the meaſure of the Angle *B* required.

Now having the knowledge of two Angles, the other may be eaſily diſcovered, being the complement of thoſe two to 180 as aforeſaid: And the Angles being known, the other ſide may be alſo found by the Probleme aforegoing.

PROBL. 3.

By the knowledge of two ſides and the Angle included, to find the other two Angles and the third ſide.

IF the Angle included be a right Angle, this is the *Proportion:* As the greater ſide is to the leſſe, ſo is the Tangent of 45 *d.* 0 *m.* to the Tangent of the leſſer Angle:

Illuſtration by numbers.

As Side A B. 230.	2.36172
To Side A D. 143.72.	2.15751
So Tang. 45.	10.00000
To Tang. A B D. 32d.	9.79579

And therefore

Extend the Compaſſes upon the Line of Numbers downwards from the greater to the leſſe ſide: then if you apply that extent upon the Line of Tangents the ſame way from 45 d. *the movable point will fall upon the Tangent of the leſſer Angle.*

Example

Example, In the Rectangle triangle *A BD* of the Diagram aforegoing, the side *A B*, being 230, and the side *A D*, 143.72 the Angle *B* will bee found 32 *d.* 0 *m.* For, if you extend the Compasses downwards upon the Line of Numbers from 230 to 143.72, that extent being applyed the same way from 45 *d.* at the top of the Line of Tangents, will cause the movable point to fall upon 32 *d.* 0 *m* *viz.* the measure of the Angle *B*, whose complement 58 *d.* 0 *m.* is the measure of the Angle *D*: And now the three Angles being thus discovered, the third side may also be known by the first Probleme of this Chapter.

But if the included Angle be oblique, *viz.* either obtuse or acute, then this is the *Anology*: As the sum of the sides known, is to the difference of the same sides: so is the Tangent of the half sum of the Angles unknown, to the Tangent of half their difference:

Illustration by numbers.

DB.	271	Ang. BDC.	122
DC.	100	Comp.	58
Sum.	371	Co. arith.	7.43063
Differ.	171		2.23299
t. $\frac{1}{2}$ Comp.	29		9.74375
t. $\frac{1}{2}$ Diff.	14.20		9.40737

Sum. 43.20 BCD.
Diff. 14.40 CBD.

And therefore.

Extend the Compasses upon the Line of Numbers downwards, and outright from the summe of the given sides, unto their difference: then applying that extent upon the Line of Tangents from the half summe of the Angles unknown, the movable point will fall upon the Tangents of half their difference, which being added unto the said half sum, makes up the greater, but being deducted from it discovers the lesser of the Angles you look for.

An *example* of this Probleme, when the moity of the Angles opposed exceeds not 45 *d.*

In the Triangle *B C D* the side *D, B*, being 271, the side *D C*, 100, and the Angle *D*, 122 *d.* the angle *B* will be found 14 *d.* 40 *m.* and the angle *C*, 43 *d.* 20 *m.* For, if you extend the Compasses upon the Mean Line of Numbers downwards from 371 (the sum of the sides known) to 171 (their difference) that extent being applied the same way upon the Line of Tangents from 29 *d.* (half the sum of the Angles *B* and *C*) the movable point will fall upon 14 *d.* 20 *m.* which being added to 29 *d.* amounts to 43 *d.* 20 *m.* for the Angle *C*, and being substracted out of them, the remainder is 14 *d.* 40 *m.* for the Angle *B.*

Two other *examples* of this Probleme, when the moity of the Angles opposed exceeds 45 *degr.*

1. In

1. In the ſame Triangle *C B D*, the ſide *C B*, being 335, the ſide *C D* 100, and the Angle *C* 43 *d.* 20 *m.* the Angle *D* will be 122 *d.* and the Angle *B* 14 *d.* 40 *m.* For if you extend the Compaſſes upon the Line of Numbers downwards from 435 (the ſum of the ſides known) to 235 (their difference) that extent being applied upon the Line of Tangents backward (*viz.* upwards) from 68 *d.* 20 *m.* (the half ſumme of the Angles *D* and *B* required) the movable point will fall upon 53 *d.* 40 *m.* which being added to 68 *d.* 20 *m.* their ſumme is 122 *degr.* 0 *min.* *viz.* the meaſure of the Angle *D*, and being deducted out of the ſame 68 *d.* 20 *m.* the remainder is 14 *d.* 40 *m.* the Angle *B*.

2. The ſide *B C*, being 335, the ſide *B D* 271, and the Angle *B* 14 *d.* 40 *m.* I demand the Angles *D* and *C* : the ſum of the ſides *B C*, and *B D*, is 606, their difference is 64, and the Angle *C* being 14 *d.* 40 *m.* the ſumme of the Angles oppoſed and unknown is 165 *d.* 20 *m.* and half that is 82 *d.* 40 *m.* Now to ſatisfie this demand, I extend the Compaſſes upon the Line of Numbers downwards from 606 to 64: then, becauſe if I apply that extent upon the Line of Tangents backwards (*viz* upwards, as before) from 82 *d.* 40 *m.* the movable point will fall as far beyond the top of that Line, as the term I look for is ſituate

tuate on this side, I apply that extent downwards from 45 *d.* 0 *m.* causing the movable point also to fall upon the same Line: that done, and the movable point remaining there fixed, I close the Compasses till the other point may rest upon 82 *d.* 40 *m.* And having the Compasses so extended, if applying that extent downwards, I set one of the points at 45 *d.* the other will reach to 39 *d.* 20 *m.* which being added to 82 *d.* 40 *min.* amounts to 122 *degr. viz.* the Angle *D*: but being deducted out of 82 *d.* 40 *m.* the remainder is 43 *d.* 20 *m.* *viz.* the measure of the Angle *C*.

And in these three cases having discovered the three Angles, the other side may be likewise found by the first Probleme of this Chapter: Observe also, that these two last *examples* will not admit of *crosse-work*: and therefore are exceptions to the general Rule delivered in the end of the same Probleme.

PROBL. 4.

The three Sides being known, to finde the Perpendicular, and the three Angles.

THe greatest side being assigned for the Base, upon which the perpendicular shall be supposed to fall, finde the summe and the difference of the other sides: that done, the *Proportion* will be this; As the Base is to the

summe

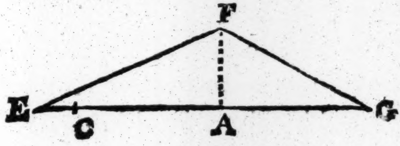
ſumme of the other ſides, ſo is the difference of the other ſides to a fourth number; which being deducted out of the Baſe, the perpendicular will fall in the middle of that which remains:

Illuſtration by Numbers.

Side EF	13	EG 20	
Side FG	11		
Sum.	24		
Differ.	2		
As Baſe EG	20.	co ar.	8.69897
To Sum.	24		1.38021
So Differ.	2		0.30103
To EC	24		0.38021
Diff.	17.6		
½ Diff.	8.8	the parts of the baſe between C and A.	

And therefore

Extend the Compaſſes upon the Line of Numbers from the parts of the Baſe unto the ſumme of the parts of the other ſides: this done, and that extent applyed the ſame way from the difference of the other ſides, will cauſe the movable point to fall upon a fourth number, which if you ſubſtract out of the intire Baſe, the perpendicular will fall in the middle of the remainder.



Example, in the Triangle *E F G*, the side *EF* being 13, and the side *F G* 11, and the Base *E G* 20, I demand the point of the Base, where the perpendicular ought to fall, and then the three Angles of the same Triangle: The summe of the sides is 24, and their difference is 2: I extend therefore the Compasses upon the Line of Numbers from 20 to 24; that done, in this *example* (because by the third *Corollary* of the first Probleme of the third Chapter, the numbers 20 and 2 are both represented at the same point) you may observe (without any farther search) the movable point to discover the parts of the segment EC, *viz*. 2.4, which being deducted out of 20, there remains 17.6, whose half is 8.8, which are the parts of the Base comprehended betwixt *C* and *A*, or betwixt *A* and *G*: I conclude therefore that A is the point of the Base where the perpendicular ought to fall. Now in the *Triangle* A*F G*, the sides A G, & G F being known, as also the Angle F A G (which is a right Angle by the 10 *D*. of the 1 *El*. of *Eucl*.) the Angles *G* and *F*, as also the perpendicular

E FA

F A may be found by the 1 and 2 Probleme of this Chapter. In like manner in the Triangle *E F* A, the sides *E* A, and *E F*, as also the Angle *E* A *F* being known, the Angle *E* and *F*, may be found by the 2 Probleme of this Chapter. And lastly, if you add the angles *E F* A and A *F G* together, their aggregate will make up the Angle *E F G* : and so by the knowledge of the three sides have you all the parts of that Triangle throughly resolved.

PROBL. 5.

The three sides being known, to find the Area, or superficiall content.

FRom the half sum of the three sides deduct each side, to the end you may discover the difference betwixt the said half sum and each side : that done, the *Proportions* will be as followeth ;

1. *As* 1 *is to the first difference, so is the second difference to a fourth number.*

2. *As* 1 *is to that fourth number, so is the third difference to a sixt number.*

3. *As* 1 *is to that sixt number, so is the half sum to an eight number, whose Square-root is the Area required.*

Example, the three sides of the foresaid Triangle *E F G*, being 20, 13, and 11, their sum is 44, half thereof is 22, and the differences betwixt each side and that half are 2, 9, and 11 :

Illustration

Illustration by numbers.

As.	1.	0.00000
To first Diff.	2	0.30103
So second Diff.	9	0.95424
To	18.	1.25527

2. *Operation.*

As.	1.	0.00000
To	18	1.25527
So third Diff.	11	1.04139
To	198.	2.29666

3. *Operation.*

As.	1.	0.00000
To	198.	2.29666
So $\frac{1}{2}$ Sum.	22.	1.34242
To	4356.	3.63908
Root.	66	1.81954

the Area required

And therefore the number required being a Square-root I extend the Compasses upon the Mean Line of Numbers upwards from 1 to 2: then that extent being applyed the same way from 9 (in the first part of that Line) the movable point wil fall upon 18 the fourth number: this done, and the movable point remaining there fixed, close the Compasses, till the other point fall again upon 1 : for that extent being applyed from 11, will cause the movable point to fall upon 198, the fixt number: again, the movable point remaining there fix-

ed, as before, open the Compasses till the other point may yet again fall upon 1, and may intercept between the legs the distance betwixt 1, and 198 : for that done, if you apply the same extent (in the first part of the same Line) from 22, the movable point will fall upon 4356, whose Square-root (by the 12. Probl. of the last Chapter) will appear at the same point upon the Great Line of Numbers to be 66, which is also the *Area* required.

2. Of Sphericall Rectangle Triangles.

Probl. 6.

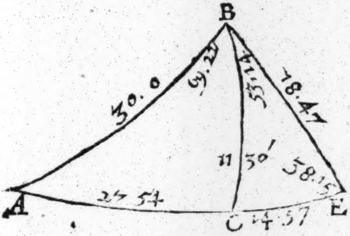
The two sides being given, to find the Base.

In Sphericall Rectangle Triangles, the side which subtends the right Angle, is called the *Base*, which to find by the knowledge of the other sides, use this *Analogie* following :

As the *Radius* or Sine of 90 *d*. is to the Sine of the complement (otherwise called the Co-sine) of one of the sides : so is the Co-sine of the other side to the Co-sine of the *Base* : And therefore

Extend the Compasses downwards upon the Line

Line of Sines from 90 d. to the Co-ſine of one of the ſides : then applying that extent the ſame way from the Co-ſine of the other ſide, the movable point will reſt upon the Co-ſine of the Baſe required.



Example, In the Triangle *A B C*, the ſide *AC*, being 27 *d.* 54 *m.* and the ſide *C B*, 11 *d.* 30 *m.* the Baſe *B* A wil be found 30 *d.* 0 *m.* for if you extend the Compaſſes downwards from 90 *d.* to 62 *d.* 6 *m.* (the complement of 27 *d.* 54 *m.*) and after apply that extent the ſame way from 78 *d.* 30 *m.* (the complement of 11 *d.* 30 *m.*) the movable point will fall upon 60 *d.* being the complement of 30 *d.* the *Baſe* required.

Illuſtration by numbers.

As Radius.		10. 00000
Co-ſine A C.	27. 54.	9. 94633
Co-ſine B C.	11. 30.	9. 99119
Co-ſine A B.	30.	9. 93752

PROBL. 7.

The two sides being known to find either of the oblique Angles.

AS the Sine of the side next the Angle required is to the *Radius* : so is the Tangent of the opposite side to the Tangent of the same Angle : And therefore,

1. When the side opposed to the Angle required exceeds not 45 *d.* *Extend the Compasses upon the Line of Sines from the Sine of the side adjacent to the Angle required, to* 90 d. *then that extent being applied the same way upon the Line of Tangents, from the Tangent of the side opposed to the required Angle, the movable point will fall upon the Tangent of the same required Angle.*

1. *Example*, In the said Triangle A B C, the side A C being 27 *d.* 54 *m.* and the side C B 11 *degr.* 30 *m.* I demand the Angle A. Extend the Compasses upon the Line of Sines from 27 *d.* 54 *m.* to 90 *d.* then that extent being applied the same way upon the Line of Tangents from 11 *d.* 30 *m.* the movable point will rest upon 23 *d.* 30 *m.* the Angle A required.

Illustration by numbers.

Sine	A C.	27. 54	9.67018
Radius		90	10.00000
Tang.	B C.	11.30	9.30846
Tang.	A.	23. 30	9.63828

Or otherwise thus :

Extend

Extend the Compaſſes acroſſe from 27 *d.* 54 *m.* upon the Line of Sines to 11 *d.* 30 *m.* upon the Line of Tangents : then applying that extent the ſame way from 90 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at a point repreſenting 23 *degr.* 30 *min.* as before. And *note*, that in this caſe the term required will always fall out to be leſſe then 45 *d.*

2. *Example*, To know the Angle *B* : Extend the Compaſſes upon the Line of Sines from 11 *d.* 30 *m.* to 90 *d.* then (becauſe that extent being applied upon the Line of Tāgents the ſame way from 27 *d.* 54 *m.* will cauſe the movable point to fall as far beyond the top of that Line, as the term required is ſituate on this ſide) apply the ſame extent backwards upon the Line of Tangents from 45 *degrees*, cauſing the movable point to fall alſo upon the ſame Line : for that done, and the movable point remaining fixed at the point where it falls, cloſe the Compaſſes till the other point may fall upon 27 *d.* 54 *m.* And at laſt that extent being applied outright upon the Line of Tangents from 45 *degr.* will cauſe the movable point to reſt upon 69 *d.* 21 *m.* the Angle *B* required. Or otherwiſe : Extend the Compaſſes acroſſe from 11 *d.* 30 *m.* upon the Line of Sines to 27 *d.* 54 *m.* upon the Line of Tangents : then if you apply that extent back-

wards from 90 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at a point representing 69 *d.* 21 *m.* as before. And here the required Angle is always greater then 45 *d.*

2. When the side opposed to the Angle required exceeds 45 d. *Extend the Compasses upon the Line of Sines from the Sine of the side adjacent to the angle required, to* 90 d. *That done, if you apply that extent backwards upon the Line of Tangents from the Tangent of the side opposed to the said required angle, the movable point will fall upon the Tangent of the same angle.*

Example, In the *Diagram* annexed, the side AC being 61 *d.* 53 *m.* and *B* C 54 *d.* 28 *m.* the angle A will be found 57 *deg.* 47 *m.* For, the Compasses being extended upon the Line of Sines from 61 *d.* 53 *m.* to 90 *d.* and that extent applied backwards upon the Line of Tangents from 54 *d.* 28 *m.* the movable point will fall upon 57 *d* 47 *m.* the Angle A required. And here *observe*, 1. that in *examples* of this kinde you cannot work acrosse: 2. the Angle here found is always greater then 45 *d.*

Probl.

PROBL. 8.

The Base and one of the oblique Angles being given, to find the other oblique Angle.

AS the *Radius* to the Co-ſine of the *Baſe*; ſo is the Tangent of the Angle known to the Co-tangent of the Angle required: And therefore,

1. When the Angle given exceeds not 45 *degr. Extend the Compaſſes upon the Line of Sines from* 90 d. *to the Co-ſine of the Baſe: then, if you apply that extent the ſame way upon the Line of Tangents from the Tangent of the Angle given, the movable point will fall upon the Co-tangent of the required Angle.*

Example, In the *Diagram* of the ſixt Probl. the Baſe A B, being 30 *d.* and the Angle A 23 *d.* 30 *m.* the Angle B will be found 69 *d.* 22 *m.* For if the Compaſſes be extended upon the Line of Sines from 90 *d.* to 60 *d.* (the complement of the Baſe) and that extent applyed the ſame way upon the Line of Tangents from 23 *d.* 30 *m.* the movable point will reſt upon 20 *d.* 38 *m.* whoſe complement (found alſo at the ſame point) is 69 *d.* 22 *m.* the Angle B required. Or otherwiſe by croſſe-work, thus: Extend the Compaſſes from 90 *d.* upon the Line of Sines to 23 *d.* 30 *m.* upon the Line of Tangents

Tangents: then that extent being applyed the same way from 60 *d.* upon the Line of Sines, the movable point will fall upon the Line of Tangents at the point representing 20 *d.* 38 *m.* as before. And here *observe*, that (in this case) the Angle you look for is always less then 45 *d.*

Illustration by numbers.

Radius	10. 00000
Co-sine A B 39	9. 93752
Tang. A 23.30.	9. 63830
Co-tang. A D C 69. 22	9. 57582

2. When the Angle given is greater then 45 *d. Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Base: this done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the Angle given, the movable point will fall upon the Co-tangent of the Angle required.*

1. *Example*, In the *Diagram* of the sixt Probl. B A, being 30 *d.* and the Angle B 69 *d.* 21 *m.* the Angle A will be found 23 *d.* 30 *m.* For if the Compasses be extended upon the Line of Sines from 90 *d*, to 60 *d.* and that extent applyed backwards upon the Line of Tangents from 69 *d.* 21 *m.* the movable point will fall upon 66 *d.* 30 *m.* the complement of 23 *d.* 30 *m.* the Angle A required. And in this case you cannot use crosse-work, and the last term found upon the Rule is always greater then 45 *d.* but the term required lesse.

2 *Example*

2. *Example*, In the *Diagram* produced in in the last Probl. B A being 74 *d.* 6 *m.* and the Angle B 66 *d.* 30 *m.* the Angle A will be found 57 *d* 47 *m.* For if you extend the Compasses upon the Line of Sines from 90 *d.* to 15 *d.* 54 *m.* & then (because that extent being applyed backwards, as before, upon the Line of Tangents from 66 *d.* 30 *m.* will cause the movable point to fall beyond that Line) if you proceed as you were directed in the second *example* of the said last Probleme at last the movable point will rest upon 32 *d.* 13 *m.* the complement of the Angle A required. Or otherwise by crosse-work : Extend the Compasses from 90 *d.* upon the Line of Sines to 66 *d.* 30 *m.* upon the Line of Tangents : this done, if you apply that extent backwards from 15 *d.* 54 *m.* upon the Line of Sines, the movable point will rest upon the Line of Tangents at the point representing 32 *d.* 13 *m.* as before. And (in this case) the last term found upon the Rule is always lesse then 45 *d.* but the term required greater.

PROBL. 9.

The Base and one of the oblique Angles being known, to find the side adjacent to the same Angle.

AS the *Radius* is to the Co-sine of the Angle known ; so is the Tangent of the Base

Base to the Tangent of the side required: And therefore,

1. When the Base is lesse then 45 d. *Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known: then applying that extent the same way upon the Line of Tangents from the Tangent of the Base, the movable point will fall upon the Tangent of the side required.*

So in the *Diagram* of the sixt Probleme B A, being 30 *d.* and A 23 *deg.* 30 *min.* the side A C (whether you work outright or acrosse) will be found 27 *d.* 54 *min.* And in this case the term required is always lesse then 45 *d.*

2. When the Base exceeds 45 *d. Extend the Compasses upon the Line of Sines from* 90 d. *to the Co-sine of the Angle known, as before: that done, if you apply the same extent upon the Line of Tangents backwards from the Tangent of the Base, the movable point will rest upon the Tangent of the side required.*

So in the *Diagram* produced in the seventh Probleme *B* A, being 74 *d.* 6 *m.* and the Angle A 57 *d.* 47 *m.* the side A C, will be found 61 *d.* 53 *m.* And in this case you cannot work acrosse, and the side to be found will be alwaies greater then 45 *d.*

Now if in applying the extent of the Compasses from the Tangent of the Base, the movable point falls beyond the Line, work as you

were

were before directed in the second *example* of the seventh Probleme aforegoing, and so shall you also in that case discover the side you look for, which will then alwaies happen to be less then 45 *d.*

PROBL. 10.

The Base *and one of the oblique Angles being known, to find the side opposed to the same Angle.*

AS the *Radius* is to the Sine of the *Base*, so is the Sine of the Angle known to the Sine of the side required: And therefore,

Extend the Compasses upon the Line of Sines from 90 d. *to the Sine of the Base: for, that extent being applied the same way from the Sine of the given Angle, will cause the movable point to fall upon the Sine of the side required.*

Example, In the *Diagram* of the sixt Probleme; to know the side B C, extend the Compasses upon the Line of Sines from 90 *d.* to 30 *d.* then if you apply that extent the same way from 23 *d.* 30 *m.* the movable point will fall upon 11 *d.* 30 *m.* the side required.

Illustration by Numbers.

Radius		10,00000
Sine AB	30	9.69897
Sine A	23.30	9.60070
Sine *B* C	11.30	9.29967

PROB.

PROBL. II.

One of the ſides and the oblique Angle next unto it being known, to find the Baſe.

AS the Co-ſine of the Angle known is to to the *Radius* ; ſo is the Tangent of the ſide given to the Tangent of the Baſe: And therefore.

1. When the ſide given exceeds not 45 *d.* *Extend the Compaſſes upon the Line of Sines from the Co-ſine of the Angle given, unto* 90 d. *This done, and that extent applyed the ſame way upon the Line of Tangents from the Tangent of the ſide given, will cauſe the movable point to fall upon the Tangent of the Baſe.* So in the *Diagram* of the ſixth Probl. the Angle A being 23 *d.* 30 *m.* and the ſide A C, 27 *d.* 54 *m.* the Baſe B A, will be found 30 *d.* 0 *m.* But here, if the movable point chance to fall beyond the Line, proceed as you have been before directed in the ſecond *example* of the 7. Probl. And in that caſe the term required will alwayes, prove greater then 45 *d.*

Illuſtration by numbers.

Co-ſine. A. 23. 30.	9. 96239
Radius	10. 00000
Tang. A C. 27. 54.	9. 72382
Tang. A B. 30.	9. 76143

2. When

2. When the given side exceeds 45 *d. Extend the Compasses upon the Line of Sines from the Co-sine of the angle given, unto* 90 d. *then, if you apply that extent upon the Line of Tangents backwards from the Tangent of the side given, the movable point will fall upon the Tangent of the Base*. So in the *Diagram* of the seventh Probl. the Angle A being 57 *d.* 47 *m.* and the side A C 61 *d.* 53 *m.* the *Base* *B* A will be found 74 *d.* 6 *m.* And here the term sought for is alwaies greater then 45 *d.*

PROBL. 12.

One of the sides and the oblique Angle next unto it being known, to find the other side.

AS the *Radius* is to the Sine of the side given; so is the Tangent of the Angle known to the Tangent of the side required: And therefore,

1. When the Angle given exceeds not 45 d. *Extend the Compasses upon the Line of Sines from* 90 d. *unto the Sine of the given side: this done, and that extent applied the same way upon the Line of Tangents from the Tangent of the angle known, will cause the movable point to fall upon the Tangent of the side required.* So in the *Diagram* of the sixt Probl. A C being 27 *d.* 54 *m.* and the angle A 23 *d.* 30 *m.* the side *B* C will

be

be found 11 *d.* 30 *m.* And in *examples* of this kind cross-work may be used, and the term sought for is always lesse then 45 *d.*

Illustration by numbers.

Radius		10. 00000
Sine A C	27. 54	9 67018
Tang. A	23. 30	9. 63828
Tang. B C	11. 30	9. 30846

2. When the Angle given exceeds 45 *d. Extend the Compasses as before: which done, if you apply that extent upon the Line of Tangents backwards from the Tangent of the given angle, the movable point will fall upon the Tangent of the side required.* So in the *Diagram* of the seventh Probl. B C, being 54 *degr.* 28 *min.* and the Angle *B* 66 *degr.* 30 *min.* the side A C will be found 61 *d.* 53 *min.* This *example* and the like cannot be performed by crosse-work; and here the term found is always greater then 45 *degr.* But if in applying the Compasses backwards the movable point chance to fall beyond the Line, work as you were before directed in the second *example* of the seventh Probleme of this Chapter, and then will the term required be alwaies less then 45 *d.*

PROBL.

PROBL. 13.

One of the sides and the oblique Angle next unto it being known, to find the other oblique Angle.

AS the *Radius* to the Co-sine of the given side: so is the Sine of the Angle known, to the Co-sine of the Angle required: And therefore

Extend the Compasses upon the Line of Sines from 90 d *to the Co-sine of the side given: this done, that extent being applyed the same way from the Sine of the given Angle, will reach to the Co-sine of the angle required.* So in the *Diagram* of the sixt Probleme A C, being 27 *d.* 54 *m.* and the Angle A 23 *d.* 30 *m.* the Angle B will be found 69 *d.* 22 *m.*

Illustration by numbers.

Radius	10. 00000
Co-sine. A C. 27. 54	9. 94633
Sine. A. 23. 30	9. 60070
Co-sine. B. 69. 21.	9. 54703

PROBL. 14.

One of the sides and the Angle opposed unto it being known, to find the Base.

AS the Sine of the Angle given is to the Sine of the side given: so is the *Radius* to the Sine of the Base: and therefore

Extend the Compasses from the Sine of the Angle given to the Sine of the given side : then if you apply that extent from 90 d. *the movable point will fall upon the* Sine *of the Base*. So in the *Diagram* of the sixt Probleme, A, being 23 *d.* 30 *m.* and the side B C 11 *d.* 30 *m.* the Base B A will be found 30 *d.* 0 *m.*

Illustration by Numbers.

Sine A	23.30	9.60070
Sine B C	11.30	9.29967
Radius		10.00000
Sine A B	30	9.69897

Probl. 15.

One of the sides and Angle opposed unto it being known, to find the other oblique Angle.

As the Co-sine of the side given is to the Cosine of the Angle given ; so is the Radius to the Sine of the Angle required : And therefore,

Extend the Compasses from the Co-sine of the given side, to the Co-sine of the given Angle : this done, that extent being applied the same way from the Radius, will cause the movable point to fall upon the Sine of the Angle required. So in the *Diagram* of the sixt Probleme, the side A C being 27 *d.* 54 *m.* and the Angle B 69 *d.* 21 *m.* the Angle A will be found 23 *d.* 30 *m.*

Illustra-

Illustration by Numbers.

Co-sine A C	27. 54	9. 94633
Co-sine B	69. 22	9. 54703
Radius		10. 00000
Sine A	23. 30	9. 60070

PROBL. 16.

One of the sides and the Angle opposed unto it being known, to find the other side.

AS the Tangent of the Angle given is to the Tangent of the side given; so is the Radius to the Sine of the side required: And therefore,

1. when neither the Angle nor side given exceeds 45 d. *Extend the Compasses downwards upon the line of Tangents from the Tangent of the Angle given, to the Tangent of the side given: this done, that extent being applied the same way upon the Line of Sines from* 90 d. *will reach to the Sine of the side required.*

So in the *Diagram* of the sixt Probleme, the Angle A being 23 *d* 30 *m.* and the side B C 11 *d.* 30 *m.* the side A C will be found 27 *deg.* 54 *min.*

2. When the Angle and the side given do each of them exceed 45 d. *Extend the Compasses upon the Line of Tangents upwards, from the Tangent of the angle given, to the Tangent of the side given: then, if you apply that extent backwards*

wards upon the Line of Sines from 90 d. the movable point will fall upon the Sine of the side required.

So in the *Diagram* of the seventh probleme, the Angle *B* being *66 d. 30 m.* and the side AC *61 d. 53 m.* the side BC will be found 54 *deg.* 28 *min.*

3. When the Angle is greater, and the side lesse then 45 d. *Extend the Compasses upon the Line of Tangents downwards from* 45 d. *to the Tangent of the angle given: then, if that extent be applied the same way from the Tangent of the given side, the movable point will fall upon a point which upon the Line of Sines represents the Sine of the side required.*

So in the *Diagram* of the sixt Probleme, the Angle *B* being *69 d. 21 m.* and the side AC 27 *d.* 54 *m.* the side BC will be found 11 *deg.* 30 *m.* And here observe, that *examples* of this kind may likewise be performed by crosse-work, the extent of the Compasses being applied backwards: For, having extended the Compasses acrosse from *69 d.* 21 *m.* upon the Line of Tangents to *90 d.* upon the Line of Sines, if you apply that extent backwards and acrosse from 27 *deg.* 54 *min.* upon the Line of Tangents, the movable point wil fall upon the Sine of 11 *d.* 30 *m.* the side required.

Probl.

PROBL. 17.

One of the sides and the Base being known, to find the Angle opposed to the same side.

AS the Sine of the Base is to the Radius, so is the Sine of the side known to the Sine of the Angle required: And therefore,

If you extend the Compasses from the Sine of the Base unto 90 d. *that extent being applied the same way, will reach from the Sine of the given side unto the Sine of the Angle required.* So in the *Diagram* of the sixt Probleme B A being 30 *d.* and the side B C 11 *d.* 30 *m.* the angle A will be found 23 *d.* 30 *m.*

Illustration by Numbers.

Sine A B 30	9.69897
Radius	10.00000
Sine B C 11. 30	9.29967
Sine A 23 30.	9.60070

PROBL: 18.

One of the sides and the Base being known, to find the oblique Angle adjacent unto that side.

AS the Tangent of the Base is to the Tangent of the given side; so is the Radius

wards upon the Line of Sines from 90 d. *the movable point will fall upon the Sine of the side required.*

So in the *Diagram* of the seventh probleme, the Angle *B* being 66 *d*. 30 *m*. and the side A C 61 *d*. 53 *m*. the side B C will be found 54 *deg*. 28 *min*.

3. When the Angle is greater, and the side lesse then 45 d. *Extend the Compasses upon the Line of Tangents downwards from* 45 d. *to the Tangent of the angle given: then, if that extent be applied the same way from the Tangent of the given side, the movable point will fall upon a point which upon the Line of Sines represents the Sine of the side required.*

So in the *Diagram* of the sixt Probleme, the Angle *B* being 69 *d*. 21 *m*. and the side A C 27 *d*. 54 *m*. the side B C will be found 11 *deg*. 30 *m*. And here observe, that *examples* of this kind may likewise be performed by crosse-work, the extent of the Compasses being applied backwards: For, having extended the Compasses acrosse from 69 *d*. 21 *m*. upon the Line of Tangents to 90 *d*. upon the Line of Sines, if you apply that extent backwards and acrosse from 27 *deg*. 54 *min*. upon the Line of Tangents, the movable point wil fall upon the Sine of 11 *d*. 30 *m*. the side required.

Probl.

PROBL. 17.

One of the sides and the Base being known, to find the Angle opposed to the same side.

AS the Sine of the Base is to the Radius, so is the Sine of the side known to the Sine of the Angle required: And therefore, *If you extend the Compasses from the Sine of the Base unto* 90 d. *that extent being applied the same way, will reach from the Sine of the given side unto the Sine of the Angle required.* So in the *Diagram* of the sixt Probleme B A being 30 *d.* and the side B C 11 *d.* 30 *m.* the angle A will be found 23 *d.* 30 *m.*

Illustration by Numbers.

Sine A B 30	9.69897
Radius	10.00000
Sine B C 11. 30	9.29967
Sine A 23 30.	9.60070

PROBL: 18.

One of the sides and the Base being known, to find the oblique Angle adjacent unto that side.

AS the Tangent of the Base is to the Tangent of the given side; so is the Radius

to the Co-ſine of the Angle required : And therefore,

1. When neither the Baſe nor the ſide given exceeds 45 d. *the extent from the Tangent of the Baſe to the Tangent of the ſide given, being applied the ſame way, will reach from 90* d. *to the Co-ſine of the angle required.*

So in the *Diagram* of the ſixt Probleme, the Baſe *B* A, being 30 *d.* and the ſide A C, 27 *d.* 54 *m.* the Angle *A* will be found 23 *d.* 30 *m.* And in this caſe croſſe-work may alſo be uſed, if you apply the Compaſſes the ſame way they were extended.

Illuſtration by Numbers.

Tang. A *B*. 30.	9.76143
Tang. A C. 27.54.	9.72382
Radius.	10.00000
Co-ſine. A. 23.30.	9.96239

2. When the Baſe and the ſide given do each of them exceed 45 *d.* *The extent upwards from the Tangent of the Baſe to the Tangent of the given ſide, being applyed backwards, will reach from* 90 d. *to the Co-ſine of the Angle required.*

So in the *Diagram* of the ſeventh Probleme, the Baſe *B A*, being 74 *d.* 6 *m.* and the ſide A C, 61 *d.* 53 *m.* the Angle A will be found 57 *d.* 47 *m.* Howbeit in this caſe croſſe-work hath no place.

3. When the *Baſe* is greater, and the ſide leſſe then 45 *d.* *Work as you were taught in the third*

third Rule of the sixteen Probleme aforegoing.

PROBL. 19.

One of the sides and the Base being known, to find the other side.

AS the Co-sine of the side given is to the *Radius*; so is the Co-sine of the Base to the Co-sine of the side required: And therefore,

The extent from the Co-sine of the side given to 90 d. *being applyed the same way, will reach from the Co-sine of the Base, to the Co-sine of the side required.*

So in the *Diagram* of the sixt Probleme the Base B A being 30 *d.* and the side A C, 27 *d.* 54 *m.* the side B C, will be found 11 *d.* 30 *m.*

Illustration by numbers

Co-sine *A* C. 27. 54	9.94633
Radius.	10.00000
Co-sine. A B. 30.	9.93752
Co-sine. B C. 11.30.	9.99119

PROBL. 20.

The two oblique Angles being known, to find the Base.

AS the Tangent of one of the Angles is to the Co-tangent of the other Angle; so is

is the *Radius* to the Co-sine of the Base: And therefore.

1. When one of the Angles given, and the complement of the other are each of them lesse then 45 *d. The extent from the Tangent of the Angle lesse then* 45 d. *unto the Co tangent of the other, will reach from* 90 d. *to the Co-sine of the Base.* So in the *Diagram* of the sixt Probleme the Angle A being 23 *d.* 30 *m.* and the Angle B 69 *d.* 21 *m.* the *Base B* A, will be found 30 *d.* And here crosse-work may likewise be used.

Illustration by numbers

Tang. A. 23. 30.	9. 63828
Co-tang. *B.* 69. 22.	9. 57582
Radius.	10. 00000
Co-sine. A *B*. 30.	9. 93754

2. When one of the Angles is greater, and the complement of the other lesse then 45 *d. Proceed as you have been taught in the third Rule of the* 16 *Probleme aforegoing.*

Probl. 21.

The two oblique Angles being known, to find either of the sides.

As the Sine of one of the Angles is to the Co-sine of the other Angle: so is the *Radius* to the Co-sine of the side opposite to the Angle, whose Co-sine was taken: And therefore,

The

The extent from the Sine of one of the Angles given, to the Co-ſine of the other, being applyed the ſame way, will reach from 90 d. to the Co-ſine of the ſide oppoſed to the Angle, whoſe Co-ſine was taken.

So in the *Diagram* of the ſixt Probleme, the Angle A being 23 *d.* 30 *m.* and the Angle *B* 69 *d.* 21 *m.* the ſide A *C*, will be found 27 *d.* 54 *m.*

Illuſtration by numbers.

Sine. A. 23. 30.	9. 60070
Co-ſine. *B.* 69. 22.	9. 54703
Radius	10. 00000
Co-ſine. A *C.* 27. 54.	9. 94633

3. Of Sphericall Obliquangle Triangles.

PROBL. 22.

Two Angles and a ſide oppoſed to one of them being known, to find the ſide oppoſed to the other.

AS the Sine of the Angle ſubtended by the ſide known is to the Sine of the ſame ſide; ſo is the Sine of the Angle ſubtended by the ſide required, to the Sine of that ſide: And therefore,

The extent from the Sine of the Angle oppoſed to

to the side known, unto the Sine of the same side, being applyed the same way from the Sine of the angle opposed to the side required, will reach to the Sine of the side so required.

So in the *Diagram* of the sixt Probleme, the Angle E, being 38 *d.* 15 *m.* the side *B* A, 30 *d.* and the Angle A 23 *d.* 30 *m.* the side B E, will be found 18 *d.* 47 *m.*

Illustration by numbers.

Sine E. 38. 15. co. ar.	0. 20824
Sine A *B*. 30.	9. 69897
Sine A. 23. 30.	9. 60070
Sine *B* E. 18. 47.	9. 50791

Probl. 23.

Two sides and the Angle opposed to one of them being known, to find the Angle opposed to the other side.

As the Sine of the side subtending the Angle known is to the Sine of the same Angle; so is the Sine of the side subtending the Angle required, to the Sine of that Angle: And therefore.

The extent from the Sine of the side subtending the angle known, to the Sine of the same angle, being applyed the same way, will reach from the Sine of the side subtending the angle required, to the Sine of that angle.

So in the *Diagram* of the sixt Probleme, B A being

being 30 *d.* the Angle E 38 *d.* 15 *m.* and the side B E 18 *d.* 47 *m.* the Angle A will be found 23 *d.* 30 *m.*

Illustration by numbers.

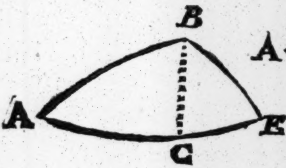
Sine AB	30.	Coar.	0.30103
Sine E	38.15		9.79176
Sine BE	18.47		9.50791
Sine A	23.30		9.60070

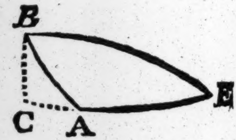
The studious Reader hath by this time (I presume) so well acquainted himself with the turnings and windings of this Instrument, *that in the resolution of most of the insuing Problemes it will (I conceive) be onely necessary to produce the bare* Analogie, *without annexing either Rule or example, as heretofore and to refer the proper applicatiou thereof to his farther industry and discretion.*

PROBL.

Probl. 24.

In any of the Triangles annexed, the sides A B, *and* A E, *together with the Angle* A, *being known, to find the side* B E.





IN an obliquangle Triangle, when the terms propounded are two ſides and one Angle, or two Angles and one ſide, and yet the term required undiſcoverable by the two laſt premiſed Problemes, you are to convert ſuch a Triangle into two Rectangle Triangles, by ſuppoſing a perpendicular to be let fall from any one of the Angles upon his oppoſite ſide, in ſuch ſort that two of the terms propounded may in one of thoſe Rectangle Triangles ſtill remain given & intire;for by this means all the other parts of ſuch a Triangle thus converted, may be readily diſcovered by the *Analogies* of Rectangle Triangles: And the perpendicular thus imagined, will fall within the Triangle, when

when the Angles adjacent to the ſide upon which it falls, are of one and the ſame kind; that is, both acute, or both obtuſe; but otherwiſe without the Triangle, when thoſe Angles are of differing kinds, *viz.* the one acute, and the other obtuſe, as plainly appears by the Triangles annexed, in which (having the ſides A B, and A E, as alſo the Angle A propounded) to find the ſide B E, uſe theſe *analogies* following.

1. As the *Radius* is to the Coſine of A; ſo is the Tangent of A B, to the Tangent of A C.

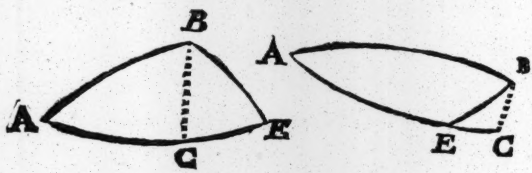
2. As the Co-ſine of A C, to the Co-ſine of C E; ſo is the Co-ſine of A B, to the Coſine of B E.

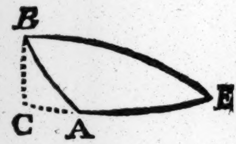
And here obſerve, that (to come to the knowledge of C E,) in caſes that reſemble the firſt of the *Diagrams* annexed, having found A C, you are to deduct it out of A E; again, in ſuch caſes as are like the ſecond *Diagram* A E, ought to be deducted out of A C; and laſtly, in thoſe that reſemble the third *Diagram* A C, and A E, are to be added together.

PROB.

PROBL. 24.

In any of the Triangles annexed, the ſides AB, *and* AE, *together with the Angle* A, *being known, to find the ſide* BE.





IN an obliquangle Triangle, when the terms propounded are two ſides and one Angle, or two Angles and one ſide, and yet the term required undiſcoverable by the two laſt premiſed Problemes, you are to convert ſuch a Triangle into two Rectangle Triangles, by ſuppoſing a perpendicular to be let fall from any one of the Angles upon his oppoſite ſide, in ſuch ſort that two of the terms propounded may in one of thoſe Rectangle Triangles ſtill remain given & intire; for by this means all the other parts of ſuch a Triangle thus converted, may be readily diſcovered by the *Analogies* of Rectangle Triangles: And the perpendicular thus imagined, will fall within the Triangle, when

when the Angles adjacent to the ſide upon which it falls, are of one and the ſame kind; that is, both acute, or both obtuſe: but otherwiſe without the Triangle, when thoſe Angles are of differing kinds, *viz.* the one acute, and the other obtuſe, as plainly appears by the Triangles annexed, in which (having the ſides A B, and A E, as alſo the Angle A propounded) to find the ſide B E, uſe theſe *analogies* following.

1. As the *Radius* is to the Coſine of A; ſo is the Tangent of A B, to the Tangent of A C.

2. As the Co-ſine of A C, to the Co-ſine of C E; ſo is the Co-ſine of A B, to the Co-ſine of B E.

And here obſerve, that (to come to the knowledge of C E,) in caſes that reſemble the firſt of the *Diagrams* annexed, having found A C, you are to deduct it out of A E; again, in ſuch caſes as are like the ſecond *Diagram* A E, ought to be deducted out of A C; and laſtly, in thoſe that reſemble the third *Diagram* A C, and A E, are to be added together.

PROB.

PROBL. 25.

In the same Triangles A B, *and* A E, *together with the Angle* A, *being known, to find either of the other Angles, and namely (for example) the Angle* E.

1. AS the Radius to the Co-sine of A; so is the Tangent of A B, to the Tangent of A C.

2. As the Sine of C E, to the Sine of A C, so is the Tangent of A, to the Tangent of E.

PROBL. 26.

A B, *and* B E, *together with* A, *being known, to finde* A E.

1. AS the Radius to the Co-sine of A; so is the Tangent of A B, to the Tangent of A C.

2. As the Co-sine of A B, to the Co-sine of B E; so is the Co-sine of A C, to the Co-sine of C E.

PROBL. 27.

A B, *and* B E, *together with* A, *being known, to find* B.

1. AS the Radius to the Co-sine of A B; so is the Tangent of A, to the Co-tangent of A B C.

2. As

2. As the Tangent of *B* E, to the Tangent of *A B*; ſo is the Co-ſine of *A B C*, to the Co-ſine of *C B E*.

PROBL. 28.

A *and* B, *together with* A B, *being known*; *to find either of the other ſides*, *and namely* (*for example*) *the ſide* B E.

1. AS the Radius to the Co-ſine of *A B*; ſo is the Tangent of *A*, to the Co-tangent of *A B C*.

2. As the Co-ſine of *C B E*, to the Co-ſine of *A B C*; ſo is the Tangent of *A B*, to the Tangent of *B E*.

PROBL. 29.

A *and* B, *together with* A B, *being known*; *to find* E.

1. AS the Radius to the Co-ſine of *A B*; ſo is the Tangent of *A*, to the Co-tangent of *A B C*.

2. As the Sine of *A B C*, to the Sine of *C B E*; ſo is the Co-ſine of *A*, to the Co-ſine of E.

PROBL. 30.

A *and* E, *together with* A B, *being known*, *to find* A E.

1 AS the Radius to the Cosine of *A*; so is the Tangent of *A B*, to the Tangent of A *C*.

2. As the Tangent of *E*, to the Tangent of A; so is the Sine of A *C*, to the Sine of *C E*.

PROBL. 31.

A *and* E, *together with* A B, *being known*, *to find B*.

1. AS the Radius to the Co-sine of A B: so is the Tangent of A to the Co-Tangent of A B *C*.

2. As the Co-sine of A, to the Co-sine of E : so is the Sine of A B C, to the Sine of C B E.

PROBL. 32.

Three sides being known, to find any of the angles.

AD the three sides together, then from the half sum thereof subtract the side opposite to the Angle required : this done, the *proportions* will be as followeth :

1. *As the Radius to the Sine of one of the sides including the angle required : so is the Sine of the other side including the same angle, to a fourth Sine.*

2. *As*

2. *As that fourth Sine is to the Sine of the half ſum of the ſides: ſo is the Sine of the difference betwixt that half ſum, and the ſide oppoſed to the angle required, to a ſeventh Sine, betwixt which and 90 d. (at the end of the Line of Sines) if you with your Compaſſes diſcover the half diſtance, that point ſhall repreſent unto you an ark, whoſe complement being doubled is the angle you look for.*

So in the *Diagram* of the ſixt Probleme the ſide A B, being 30 *d.* the ſide B E, 18 *d.* 47 *m.* and the ſide A E, 42 *degr.* 51 *min.* I demand the Angle *B*: The ſum of the ſides is 91 *d.* 38. *m.* half that ſum is 45 *d.* 49 *m.* The ſide A E, being ſubtracted out of that half, there remains 2 *d.* 58 *m.* And therefore to diſcover the Angle *B*, proceed thus:

Extend the Compaſſes upon the Line of Sines from 90 *d.* unto 30 *d.* then applying that extent the ſame way, and upon the ſame Line from 18 *d.* 47 *m.* the movable point will fall upon 9 *d.* 16 *m.* again, that point remaining there fixed, extend the compaſſes ſo far that their other point may reſt upon 45 *d.* 49 *m.* this done, and that extent applyed the ſame way from 2 *d.* 58 *m.* will cauſe the movable point at laſt to fall upon 13 *d.* 20 *m.* whoſe half diſtance towards 90 *d.* will happen upon a point repreſenting 28 *d.* 42 *m.* whoſe complement (*viz.* 61 *d.* 18 *m.*) being doubled, amounts to 122 *d.* 36 *m.* the quantity of the Angle *B* required.

Illustration by Numbers.

Radius.	10.00000
Sine. A *B*. 30.	9.69897
Sine. *B* E. 18. 47.	9.50791
Fourth-Sine.	9.20688
Sine ½ sum. 45. 49.	9.85558
Sine Diff. 2. 58.	8.71395
Seventh-sine	9.36265
Radius.	10.00000
Sine. 28. 42.	9.68132

*C*omp. 61. 18. being doubled is 122. 36. The angle A *B* E.

Probl. 33.

The three Angles being known, to find any of the sides.

If in stead of the greatest Angle, you take his complement to 180 *d*. the Angles convert themselvs into sides, and the sides into Angles, and then (by consequent) the operation will be the same with that of the last *Probleme*.

4. Of divers other Geometrical Figures.

Probl. 34. *The Diameter of a Circle being known, to find the Circumference.*

The

The extent upon the Line of Numbers from 1 to the Diameter, will reach from 3. 142 to the circumference.

Probl 35. *To find the superficiall content.*

The extent from 1 to the Diameter being twice repeated from .7854 will reach to the Content. Otherwise thus: The extent upon the Great Line Numbers from 1 to the Diameter, will reach upon the Mean Line of Numbers from .7854 to the content. Or yet thus: the extent upon the Great Line of Numbers from 1 to .7854 will reach upon the Mean Line of Numbers from the Diameter to the Content. And in this manner may divers of the insuing Problemes be diversified, which (as before) I refer to the discretion of the Practioner.

Probl. 36. *To find the side of the square, which may be inscribed within the same Circle.*

The extent from 1 to .7071 will reach from the Diameter to the side of the square required.

Probl. 37. *Having the Circumference, to find the Diameter.*

The extent from 1 to .3183 will reach from the Circumference to the Diameter.

Probl. 38. *To find the superficiall Content.*

The extent from 1 to the Circumference being twice repeated from ,07958, will reach to the content. Or, &c.

Probl. 39. *To find the ſide of the ſquare, which may be inſcribed within it.*

The extent from 1 to the circumference, will reach from .2251 to the ſide of the ſquare required.

Probl. 40. *Having the Content of a Circle, to find the Diameter.*

The extent from 1 to 1. 273 will reach from the content to another Number, whoſe ſquare-root is the Diameter required.

Probl. 41. *To find the Circumference.*

The extent from 1 to 12.57 will reach from the content to another number, whoſe ſquare-root is the circumference required.

Probl. 42. *To find the ſide of the ſquare equall unto it.*

Extract the ſquare-root thereof [by the 12 *Probl.* of the laſt Chapter, and you have your deſire.

Probl. 43 *The breadth of a long ſquare being given in Inch-meaſure, and the length in Foot-meaſure, to find the Content in feet.*

The extent from 12 to the breadth in inches, will reach from the length in feet to the content in feet. Or, *vice verſa*, the extent from 12 to the length in feet, will reach from the breadth in inches to the content in feet.

Probl. 44. *The breadth and length of a long ſquare being given in Foot-meaſure, to find the Content thereof in yards.*

The

The extent from 9 to the breadth, will reach from the length to the Content in yards. Or, &c.

Probl. 45. *To find the Content in single Perches.*

The extent from 16. 5 to the breadth, will reach from the length to the content in single Perches. Or, &c.

Probl. 46. *To find the Content in square-perches, otherwise* (*in* Architecture) *called* Poles.

The extent from 272. 25 to the breadth, will reach from the length to the content in Poles. Or, &c.

Probl. 47. *The breadth and length of a long square being given in Perches, to find the content in Acres.*

The extent from 160 to the breadth, will reach from the length to the content in Acres. Or, &c.

Probl. 48. *The breadth and depth of a square Rectangle solid, being given in Inch-measure, and the length in Foot-measure, to find the content thereof in Feet.*

The extent from 12 to the breadth or depth in Inches, being twice repeated from the length in feet, will reach to the content in Feet, Or, &c.

Probl. 49. *The breadth and depth of a Rectangle solid (not just square) being known in*

Inch-measure, and the length in foot measure to find the content in Feet.

Find (by the tenth *Probleme* of the last Chapter) the mean proportional betwixt the breadth and the depth; for then, the extent from 12 to that Mean Proportional, being twice repeated from the length in feet, will reach to the content in Feet.

Probl. 50. *The breadth and depth of a Rectangle solid (not just square) being known in Foot-measure, to find the Base or superficies at the end thereof.*

The extent from 1 to the breadth, will reach from the depth to the Base required.

Probl. 51. *The Base and length of a Rectangle solid being known in Foot-measure, to find the content in Feet.*

The extent from 1 to the Base, will reach from the length to the content.

Probl. 52. *Having the Diameter of a Cylinder, to find the Base.*

The Base of a Cylinder being a perfect Circle, this *Probleme* may be resolved by the 35. aforegoing.

Probl. 53. *The Base and length of a Cylinder being known, to find the content.*

The extent from 1 to the Base, will reach from the length to the content.

Probl. 54. *Having the* Axis *of a Sphere, to find the superficial content.*

The

The extent from 1 to the *Axis*, being twice repeated from 3. 142, will reach to the superficial content required. Or, &c.

Probl. 55. *To find the solid Content.*

The extent from 1 to the *Axis*, being thrice repeated from .5238, will reach to the solid Content required. Or, &c.

Cap. VI.

The use of the Rule of Proportion in Astronomie.

Probl. 1.

By the Suns shadow, to find his height.

THe extent upon the Mean Line of Numbers, from the length of the Rules shadow to the height thereof (held perpendicular to the Horizon) will reach upon the Line of Tangents from 45 *d.* to the Suns height required.

Probl. 2. *The Suns greatest declination, together with his distance from the next Equinoctiall point being known, to find his present declination.*

As the *Radius* to the Sine of the Suns distance from the next Equinoctiall point; so is the Sine of the Suns greatest declination to the Sine of the declination required.

Probl. 3. *To find the right ascention.*

As the *Radius* to the Tangent of his distance, &c. so is the Co-sine of his greatest declination to the Tangent of his right Ascention.

Probl. 4. *The Suns greatest declination, together with his present declination, being known, to find his right Ascention.*

As the Tangent of his greatest declination to the *Radius*, so is the Tangent of his present declination to the Sine of his right Ascention.

Probl. 5. *The elevation of the Pole, together with the Suns declination being known, to find how long the Sun riseth or setteth before or after the hour of six.*

As the Co-tangent of the Elevation is to the *Radius*; so is the Tangent of the Suns declination to the Sine of the *ascensional* difference between the hour of six, and the Suns rising or setting.

Probl. 6. *To find the Suns amplitude.*

As the the Co-sine of the Elevation is to the Sine of the Declination; so is the *Radius* to the Sine of the Amplitude.

Probl. 7. *The Elevation of the Pole, the Suns greatest declination, and his distance from the next Equinoctial point being known, to find the Amplitude.*

As the Co-sine of the Elevation is to the Sine of the Suns distance; so is the Sine of the

Suns

Suns greatest declination to the amplitude required.

Probl. 8. *When the Sun is in the Equinoctial, by knowing the Elevation of the Pole, to find the Suns height at any time assigned.*

As the *Radius* to the Cosine of the Elevation; so is the Sine of the Suns distance from six a clock to the Sine of the height required.

Probl. 9. *The Elevation of the Pole, and the declination of the Sun being known, to find the Suns height at the hour of six.*

As the *Radius* to the Sine of the Latitude; so is the Sine of the Declination to the Sine of the height required.

Probl. 10. *To find the Suns height at any time assigned.*

1 As the Radius to the Co-tangent of the Elevation; so is the Sine of the Suns distance from six, to the Tangent of an Ark, which being subtracted out of the Suns distance from the Pole, I say again,

2. As the Co-sine of the Ark found is to the Co-sine of the residue of the Suns distance from the Pole; so is the Sine of the Elevation to the Sine of the height required.

Probl. 11. *To find the time when the Sun will be due East and West.*

As the Tangent of the Elevation to the Radius; so is the Tangent of the Declination to the Co-sine of the hour from the Meridian.

Probl.

Probl. 12. *To find the Suns height, when he cometh to be due East and West.*

As the Sine of the Elev. to the Radius; so is the Sine of the declin. to the height required.

Probl. 13. *To find the Suns Azimuth at the hour of six.*

As the Co-sine of the Elevation is to the Co-tangent of the Declination; so is the Radius to the Tangent of the Azimuth from the North part of the Meridian.

Probl. 14. *The complement of Elevation, the Suns distance from the Pole, and the complement of the Suns height being known, to find the Azimuth.*

Having added the three given terms together, find the difference betwixt their half sum and the Suns distance from the Pole; this done, the *Proportions* will be as followeth:

1. As the Radius to the Co-sine of the Elevation; so is the Co-sine of the height to a fourth Sine:

2. As that fourth Sine is to the Sine of the half sum; so is the Sine of the difference to a seventh Sine, whose half distance towards 90 *d.* will discover the Sine of an Arke, whose complement being doubled is the A*zimuth* you look for.

Probl. 15. *To find the hour of the day.*

Having added the three given terms together, as before, find the difference betwixt their half

half ſum and the complement of the Suns height : this done, the *Proportions* will be theſe.

1. As the Radius to the Co-ſine of the Elevation; ſo is the Sine of the Suns diſtance from the Pole to a fourth Sine.

2 As that fourth Sine is to the Sine of the half ſum : ſo is the Sine of the difference to a ſeventh Sine, whoſe half diſtance towards 90 *d.* will diſcover the Sine of an Ark, whoſe complement being doubled and converted into time, will produce the hour required.

CAP. VII.

The uſe of the Rule of Proportion in Dialling.

Probl. 1. *To make a direct Polar Diall.*

HAving aſſigned a Line drawn in the middle of the Plane for the Meridian, and another Line drawn parallel unto it for ſome other hour, which may be deſcribed upon the Plane : I ſay,

1. As the Tangent of that hour is to the Radius; ſo is the diſtance of that hour-line from the Meridian to the height of the ſtile.

2. As the Radius is to the height of the ſtile; ſo is the Tangent of any other hour, to the diſtance

diſtance of the ſame hour from the ſubſtile.

Probl. 2. A *Meridian Diall.*

Having drawn a Line repreſenting part of the *Axis* of the world towards a proper ſide of the Plane,(according to his ſituation either Eaſtward or Weſtward) and aſſigned that Line for the hour of ſix, the *Proportions* wil fal out to be as in the former *Probleme*; for,

1. As the Tangent of any hours diſtance from ſix is to the Radius; ſo is the diſtance of the hour upon the Plane from the Hour-line of ſix. to the height of the ſtile.

2. As the Radius is to the height of the ſtile; ſo is the Tangent of any other hours diſtance from ſix, to the diſtance of the ſame hour from the ſubſtile.

Probl. 3. *An horizontall Diall.*

As the Radius to the Tangent of the hour given: ſo is the Sine of the Elevation to the Tangent of the hour-line from the Meridian.

Probl. 4. *A verticall Diall.*

As the Radius to the Tangent of the hour: ſo is the Co-ſine of the Elevation to the Tangent of the Hour-line from the Meridian.

Probl. 5. *A verticall Inclining Diall.*

Having found out the Elevation of the Pole above the Plane, according to its inclination, the Proportion, will be this:

As the Radius to the Tangent of the Hour: ſo is the Sine of the Elevation above the Plane,

to

to the Tangent of the Hour-line from the Meridian.

Probl. 6. *A vertical Declining Diall.*

1. As the *Radius* to the Co-tangent of the Elevation : so is the Sine of the Declination to the Tangent of the Substiles distance from the Meridian of the place.

2. As the Radius to the the Co-sine of the Declination : so is the Co-sine of the Elevation to the Sine of the Stiles height above the Substile.

3. As the Sine of the Elevation is to the Radius, so is the Tangent of the Declination to the Tangent of the Inclination of the Meridian of the Plane to the Meridian of the Place.

4. As the Radius to the Sine of the Stiles height above the Substile : so is the Tangent of the Angle at the Pole comprehended between the hour given and the Meridian of the Plane, to the Tangent of the Hour-lines distance from the Substile.

Probl. 7. *A Meridian Inclining Dial.*

1. As the Radius to the Tangent of the Elevation : so is the Sine of the Inclination to the Tangent of the Substiles distance from the Meridian.

2. As the Radius is to the Sine of the Elevation : so is the Co-sine of the Inclination to the Sine of the stiles height above the Substile.

As

3. As the Co-ſine of the Elevation is to the Radius: ſo is the Tangent of the Inclination, to the Tangent of the Inclination of Meridians.

4. As the Radius is to the Sine of the Stiles height above the Subſtile: ſo is the Tangent of the Angle at the Pole, to the Tangent of the Hour-lines diſtance from the Subſtile.

Probl. 8. *A Polar Declining Diall.*

1. As the Radius to the Sine of the Declination: ſo is the Co-ſine of the Elevation to the Coſine of the Ark comprehended between the Horizon and the Subſtile.

2. As the Radius to the Tangent of the Declination: ſo is the Sine of the *E*levation to the Tangent of the Inclination of Meridians, which being converted into time, ſheweth how many hours the *S*ubſtile ought to be placed from the Hour-line of 12.

3. As the Radius is to the Tangent of the hours diſtance from the *S*ubſtile: ſo are the parts of the height of the *S*tile, to the diſtance of the *S*ubſtile from the Hour-line required, meaſured by a *S*cale of like parts.

Probl. 9. *A Declining Inclining Diall.*

1. As the Radius to the Tangent of Inclination to the Horizon: ſo is the Co-ſine of Declination to the Tangent of the Ark of the Meridian of the place intercepted between the Horizon and the Plane, which being compa-

red

red with the Elevation of the Pole, the distance of the Pole from the Plane may be thereby readily discovered.

2. As the Radius is to the Sine of the Declination from the Vertical : so is the Sine of Inclination to the Horizon, to the Co-sine of the Inclination to the Meridian.

3. As the Radius is to the Co-sine of Inclination to the Horizon : so is the Co-tangent of Declination to the Tangent of the Ark of the Plane intercepted between the Horizon and the Meridian of the Place.

4. As the Radius is to the Sine of the inclination to the Meridian : so is the Tangent of the Poles distance from the Plane, to the Tangent of the Substiles distance from the Meridian.

5. As the Radius is to the Poles distance from the Plane : so is the Sine of the Inclination to the Meridian, to the Sine of the Stiles height above the Substile.

6. As the Co-sine of the Poles distance from the Plane is to the Radius : so is the Co-tangent of the Inclination to the Meridian, to the Tangent of the Inclination of Meridians.

7. As the Radius is to the Stiles height above the Substile : so is the Tangent of the Angle at the Pole, to the Tangent of the Hour-lines distance from the Subtile.

Cap. VIII.

The use of the Rule of Proportion in Geographie.

Probl. 1. *Two places being propounded, which differ onely in Latitude, to find their distance.*

1. WHen the two places are situate under the same Meridian, and upon the same side of the Equinoctiall; *Subtract the lesser Latitude out of the greater; that done, the remainder is the distance required.*

2. When one of the places propounded is situate upon this side the Equinoctiall, and the other upon that, and yet both under the same Meridian, as before: *Add the two Latitudes together; this done their sum is the distance required.*

Probl. 2. *Two places, which differ onely in Longitude, being propounded, to know their distance.*

1. When the Places are both of them situate under the Equinoctiall: *Subtract the lesser Longitude out of the greater: this done, the remainder is the distance required.*

2. When the places are situate under some parallel betwixt the Equinoctiall and one of the Poles: *Then, as the Radius is to the Co-sine of*

of the common Latitude given : so is the Sine of half the difference of Longitude, to the Sine of half the distance.

Probl. 3. *Two places being given, which differ both in Longitude and Latitude, to find their distance.*

1. When one of the places is situate under the Equinoctiall, and the other towards one of the Poles: Then, *As the Radius is to the Co-sine of the difference of Longitude : so is the Co-sine of the Latitude given, to the Co-sine of the distance required.*

2. when both places are without the Equinoctiall, and towards one of the Poles: Then, *As the Radius is to the Co-sine of the difference of Longitude : so is the Co-tangent of the lesser Latitude to the Tangent of another Arke,* which being subtracted out of the complement of the greater Latitude, if the difference of Longitude be less then a quadrant, or added thereto if more, their sum or difference shall be the fift Ark; then say, *As the Co-sine of the Ark found is to the Co-sine of the fift Ark: so is the Sine of the lesser Lat. to the Co-sine of the distance required.*

3. When both places are without the Equinoctiall, and one of them situate towards the North Pole, and the other towards the South: say thus, *As the Radius is to the Co-sine of the difference of Longitude : so is the Co-tangent of one of the Latitudes, to the Tangent of another Arke,* which being subtracted out of the other

Latitude, and 90 d. added together, if the difference of Longitude be less then a quadrant, or added thereto if more, their sum or difference shall be the fift Ark; then say, *As the Co-sine of the Ark found is to the Co-sine of the Ark remaining: so is the Sine of the Latitude first taken, to the Co-sine of the distance required; if the fift Ark was subtracted, or the complement of the distance, if added to the other Latit.*

Cap. IX.

The use of the Rule of Proportion in Navigation.

Probl. 1.

The Latitudes of two places being known, to find the Meridional difference.

1. When one of the places is situate under the Equinoctial, and the other without: *The degrees and decimal Minutes found upon the Scale of equal parts at the point, where that other Latitude is represented upon the Scale of Latitudes, are the Meridional difference required.*

2. When one of the places have Southerly, and the other Northerly Latitude: *Extend the Compasses upon the Line of Latitudes, from the beginning of that Line to the lesser Latitude: that done, if you apply that extent upon the same Line, and the same way from the greater Latitude, the movable point will discover upon the Line of equal parts, the Meridional difference desired.*

3. When both places have Northerly or Southerly Latitude: *Extend the Compasses upon the Line of Latitudes from one of the Latitudes to the other: this done, if you apply that extent from the beginning of the Line, the movable point will shew you upon the Scale of Equal parts the Meridional difference you look for.*

Probl. 2. *The Latitudes of two places together with their difference of Longitude being known, to find the Rumbe directing from one to the other.*

As the Meridional difference is to the difference of Longitude: so is the Radius to the Tangent of the Rumbe: And therefore,

The extent upon the mean Line of Numbers from the Meridional difference to the difference of Longitude, will reach upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe.*

And *note* here, that in this Probleme and the like, you may make use of the double Scale, placed upon the last Line of the Rule of Proportion, at the end of the Scale of Inches: *viz.* (if need be) for the more speedy reduction of the Sexagenary Minutes of the Longitude into Decimals, *& contrà*: to the end you may by that means the more readily work by them upon the Mean Line of Numbers.

Probl. 3. *By both Latitudes and Rumbe to find the distance upon the Rumbe.*

As the Co-sine of the Rumbe to the true

difference of Latitudes : so is the Radius to the distance required : And therefore,

Extend the Compasses acrosse from the Co-sine of the Rumbe (found upon the Line of Sines) to the true difference of Latitudes (found upon the Mean Line of Numbers :) this done, if you apply that extent the same way and acrosse from 90 d. *upon the Line of Sines, the movable point will shew you upon the Mean Line of Numbers (in Degrrees and Decimal Minutes) the distance required.*

Probl. 4. *By both Latitudes and Rumbe, to find the difference of Longitude.*

As the Radius to the Tangent of the Rumb: so is the Meridional difference of the Latitudes to the difference of Longitude required: And therefore,

The extent upon the Line of Tangents from 45 d. *to the Tangent of the Rumbe, will reach upon the Mean Line of Numbers from the Meridional difference of the Latitudes to the difference of Longitude required.*

Probl. 5. *By both Latitudes and distance to find the Rumbe.*

As the distance is to the true difference of Latitudes : so is the Radius to the Co-sine of the Rumb : And therefore,

The extent upon the Mean Line of Numbers, from the distance to the difference of Latitudes, will reach upon the Line of Sines from 90 d. *to the Co-sine of the Rumb.*

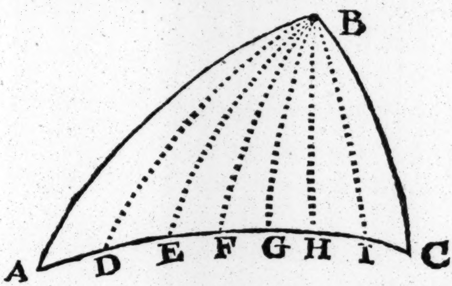
Probl.

Probl. 6. *By one Latitude, Distance, and Rumbe, to find the other Latitude.*

As the Radius to the Co-sine of the Rumb: so is the distance to the true difference of Latitudes: And therefore,

The extent upon the Line of Sines *from* 90 d. *to the Co-sine of the Rumbe, will reach upon the Mean Line of Numbers, from the distance to the true difference of Latitudes.*

Probl. 7. *The Latitudes and difference of Longitude of two places being known, to sail by the great Circle from the one to the other.*



In the Triangle A B C, let A represent *S. Christophers*, C the *Lizard*, B the North Pole, A B the complement of the Latitude of *S. Christophers, viz.* 74 *d.* 30 *m.* B C the complement of the Latitude of the *Lizard*, 40 *d.* 0 *m.* and A B C the difference of Longitude, 68 *d.* 30 *m.* Now therefore to steer a course from A to C alongst the Ark A C, proceed thus:

1. By the 24 and 25 *Problemes* of the fift

Chapter find the ſide A C, as alſo the Angles A and C.

2, By the 22 of the ſame Chapter find the perpendicular *B J*, cutting the ſide A C, at right Angles.

3. By the 8 of the ſame diſcover the Angle A B I, and by the 9 the ſide A I.

4. Leſſening the Angle A B I, two, five, or ten degrees, as you ſhall ſee cauſe (for *example*, by the Angle A B D) by the knowledge of the Angle D B I, and of the ſide B I, find by the 11, 12, and 13 *Problemes* of the ſame 5 Chapter, the Baſe B D, the ſide D I, and the Angle B D I; and ſo proceeding to do the like at the points E F G and H, you may thereby diſcover the ſeveral diſtances betwixt point and point, the ſeveral Latitudes at thoſe points, and the ſeveral Angles, according to which you are to direct your courſe: For at firſt, from A you are to ſteer according to the Angle B A I, until you ſhall have ſailed ſo many Leagues as anſwer to the diſtance betwixt A and D: and then from D, according to the Angle B D I, untill you ſhall arrive at the point E, according to the number of Leagues that D and E are diſtant the one from the other: and ſo conſequently of the reſt in their order, untill you ſhall attain the point *I*, from whence you are to ſteer full Weſt towards C, the Angle B I C being a right Angle, &c.

CAP.

CAP. X.

The use of the Rule of Proportion in the gaging of Vessell.

Probl. 1. *The true content of a solid Measure being known, to find the Gage-point of the same Measure.*

THe Gage-point of a solid Measure is the Diameter of a Circle, whose superficiall content is equall to the solid content of the same Measure: so the solid content of a Wine-gallon (according to *Winchester* measure) being 231 Cube-inches, if you conceive a Circle to contain so many inches, you shall find) by the 40 *Probleme* of the 5 Chapter) the *Diameter* thereof to be 17. 15: For,

As 1 *is to* 1. 273: *so is* 231 *to* 294. 1, *whose Square-root* (by the 12 *Probleme* of the same Chapter) *is* 17. 15, *the Gage-point of Wine-measure.*

Thus likewise may you easily discover the Gage-point of Ale-measure, an Ale-gallon (as it hath been of late discovered) containing 288 Cube-inches: For,

As 1 *is to* 1. 273 : *so is* 288 *to* 366, 7, *whose Square-root is* 19. 15, *the Gage-point of Ale-measure.*

And (indeed) 288 Cube-inches seem to be the most probable content of an Ale-gallon, being the sixt part of 1728, which is the number of Cube-inches contained in a Cube-foot. For so (according to that account) a Cube-foot contains just six Gallons, and the Gage-point of Ale-measure (by reason of the soile and wast) exceeds that of Wine-measure just two Inches.

After the same manner also may you discover the Gage-point of any forrain Measure whatsoever, and afterwards by that means come to the knowledge of the true content of their Vessell, according to the Measures used amongst them, as will plainly appear by that which shall hereafter be taught for the discovery of the Contents of Wine and Beer vessell according to the English measures.

Now from that which is abovesaid doth necessarily follow this Corollary: *When the Diameter of a Cylinder in Inches is equal to the Gage point of any measure (given likewise in Inches) every Inch in the length thereof contains one Integer of the same measure:* So in a Cylinder having 17.15 Inches diameter, every Inch in the length thereof contains one intire Wine-gallon: and in another having 19.15 inches diameter, every Inch thereof contains one Ale-gallon &c.

Probl. 2. *In a Wine or Beer vessell, the Dia-*

meters

meters at the Head and Bongue being known, to find the equated Diameter.

Extend the Compasses upon the Line of Inches from the Diameter at the Head, to the Diameter at the Bongue : then applying that extent from the begining of the same Line, and observing there the difference betwixt the two Diameters, (one of the points remaining still fixed at the beginning of that Line) close the Compasses til the other point may fal upon so many parts of the *Gage-line*, as the difference between the two Diameters amounts unto in Inches : This done, and that extent applied from the Diameter at the Head towards the Diameter at the Bongue, will cause the movable point to fall upon the *equated* Diameter you look for.

Example, The Diameter at the head being 18. 3 Inches, and that at the Bongue 21. 5 Inches, I demand the *equated* Diameter. First, extending the Compasses upon the Line of Inches from 18. 3 Inches to 21. 5. and then applying that extent from the beginning of the same Line, I find the movable point to fall upon 3. 2 Inches, *viz.* the true difference of the two Diameters : Now therefore if still keeping one of the points of the Compasses fixed at the beginning of that Line, I close them till the other point may fall at 3. 2 upon the *Gage-line*, and after apply that extent

from

from 18. 3 (the Diameter at the Head) the movable point will at last fall upon 20. 54 Inches, the *equated* Diameter required. And by this means your Vessel, which before was in part of an Oval form and irregular, is now reduced into a perfect *Cylinder*.

Probl. 3. *The Equated Diameter and length of a Wine or Beer-vessel being given in Inches, to find the content thereof in Wine-measure.*

The extent upon the *Line of Numbers* from 17. 15 (the Gage-point of Wine-measure) to the *equated* Diameter, being twice repeated from the length, will reach to the content in Wine-gallons.

Probl. 4. *To find the Content in Ale-measure.*

The extent from 19.15 (the Gage-point of Ale-measure) to the *equated* Diameter, being twice repeated from the length, will reach to the content in Ale-gallons.

Probl. 5. *Having the length and the two Diameters at the Head and Bongue, together with the equated Diameter and Content of a Vessel, out of which so much and no more of the liquor is drawn, that the superficies thereof may cut some part of the Head; to find the true quantity of the remainder.*

Deduct half the difference of the Diameters at the Head and Bongue, out of the distance intercepted between the Bongue and the superficies of the liquor, to the end you may thereby

thereby diſcover where the liquor within the Veſſel cuts the head, according to which draw a Line with Chalk (or otherwiſe) upon the Head; then having drawn another Line parallel to the firſt, and of like diſtance from the other oppoſite ſide of the Head, you have in the middle of that Head betwixt thoſe two Lines a Segment of the Veſſel marked out, and likewiſe two other Segments, the one above and the other below that middle Segment: after this, taking the length of one of thoſe parallels in Inch-meaſure, the *equated* Diameter of the ſuperficies may be thus found out upon the *Rule*.

The extent from the Diameter at the Head to the equated Diameter of the Veſſel, will reach from the length of one of the Parallels to the equated Diameter of the ſuperficies.

Then having diſcovered (by the 2 *Probleme* aforegoing) the *equated* Diameter of thoſe two other *equated* Diameters, find (by the 10 *Probleme* of the 4 Chapter) the Mean proportional between that third *equated* Diameter and the diſtance between the two Parallels: This done, make uſe of that Mean proportional, as an *equated* Diameter of the middle Segment, and then finding by one of the two laſt *Problemes* (according to the queſtion propounded) the Content thereof in Gallons, &c. deduct that Content out of the whole Content

tent of the Vessel : All this performed, *when the Vessel is above half full,* the Content of that middle Segment and half that remainder being added together, is the Content you look for. But, *when the vessel is not half full,* half that remainder is the content desired.

CAP. XI.

The use of the Rule of Proportion in Military Orders.

Probl. 1. *Any number of Souldiers being proprunded, to order them into a square battaile of men.*

FInd (by the 12 *Probleme* aforegoing) the square-root of the number given: For, look how much that root shall happen to be, so many Souldiers ought you to place in Rank, and so many likewise in File, to make a square Battaile of men.

Example, Let it be required to order 573 Souldiers into a square Battaile of men: the Square-root of that number is 23. 94: and therefore you are to place 23 in Rank, and as many also in File: For Fractions are not considerable in questions that concern *Military* Orders.

Probl. 2. *Any number of of Souldiers being propounded*

propounded, to order them into a double Battaile of men : viz. *which may have twice so many in Rank as in File.*

Find out the Square-root of half the number given : for that root is the number of Souldiers to be placed in File : and so many more ought to be placed in Rank, to make up a double Battaile of Men.

Example, 1342 Souldiers being propounded to be put into that order : I find 26, &c. to be the Square-root of 671 (half the number propounded) and thereupon conclude that 26 ought to be placed in File, and 52 in Rank, to order so many Souldiers into a double Battail of men.

Probl. 3. *Any number of Souldiers being given, to order them into a quadruple Battail* viz. *such as may have four times so many in Rank as in File.*

Here the Square-root of the fourth part of the number given will shew the number to be placed in File, and four times so many are to be placed in Rank.

So 2048 Souldiers being offered to be put into that order, 22 are to be placed in File, and 88 in Rank. For, the fourth part of 2048 is 512, whose Square-root is 22, &c.

probl. 4. *Any number of Souldiers being given, together with their distances in Rank and File to order them into a Square Battail of ground.*

Extend

Extend the Compaſſes upon the Mean Line of Numbers, from the diſtance in File to the diſtance in Rank : this done, and that extent applyed the ſame way, and upon the ſame Line from the number of Souldiers propounded, will cauſe the movable point to fall upon a fourth number, whoſe Square-root appearing at the ſame point upon the Great Line of Numbers) is the number of men to be placed in File : by which if you divide the whole number of Souldiers, the Quotient will ſhew the number of men to be placed in Rank;

Example, 2500 men are propounded to be ordered into a ſquare Battaile of ground, in ſuch ſort that their diſtance in File being ſeven foot, and their diſtance in rank three foot, the ground whereupon they ſtand may be a juſt ſquare : To reſolve this queſtion, extend the Compaſſes upon the Mean Line of Numbers downwards from 7 to 3 : then (becauſe the fourth number to be found in all likelihood will conſiſt of four figures) if you apply that extent the ſame way from 2500 in the firſt part of the ſame Line, the movable point will fall upon the fourth number you look for, where alſo you may obſerve 32, &c. upon the ſecond part of the great Line of Numbers, which are the number of men to be placed in File; again, if letting that point of the Compaſſes remain fixed there, you cloſe them

them till the other point may reach crosse-wise to 1 at the beginning of the first part of the said Great Line of Numbers, that extent being applied the same way (*viz.* downwards and acrosse) from 2500 upon the same great Line, the movable point will fall near 76, &c. which are the number of Souldiers to be placed in Rank.

Probl. 5. *Any Number of Souldiers being propounded, to order them in Rank and File according to the reason of any two numbers given.*

This Probleme is resolved much after the same manner that the last was: For,

As the proportional number given for the File is to that given for the Rank: so is the number of Souldiers to a fourth number, whose root is the number of men to be placed in Rank, by which if you divide the whole, the quotient is the number to be placed in File.

So if 2500 Souldiers were to be martialled in such order, that the number of men to be placed in File might bear such proportion to the number of men to be placed in Rank, as 5 bears to 12: I say then, as 5 is to 12, so is 2500 to another number, whose root is 77, &c. *viz.* the number of men to be placed in Rank, by which if the same 2500 be divided, the quotient will be 32, &c. the number of men to be placed in File.

CAP.

Cap. XII.

The use of the Rule of Proportion in questions that concern Interest and Annuities.

Probl. 1. *A summe of Money beeing forborn for a certaine time, to find how much it will be augmented at the expiration of the same time, accounting Interest upon Interest according to a certain rate propounded.*

THe extent upon the Line of Numbers from 100 *l*. to the aggregate of 100 *l*. and the rate added together, being repeated the same way from the sum given, so many times as there are years in the question, will at last cause the movable point to fal upon the Principall increased with the Interest, according to the forbearance and rate propounded.

Example, I desire to know how much 273 *l*. being forborn for 5 years wil be increased at the expiration of those years according to Interest upon Interest, and the rate of 8 l. *per centum*: Extend the Compasses upon the Great Line of Numbers from 100 to 108: This done if that extent be repeated five times from 273, the movable point will at last fall upon 402. 1 (*viz*. 402 *l*. 25) the Principall augmented with

with the Intereſt for the forbearance of thoſe five years.

Probl. 2. *A ſumme of Money being due at a time to come, to finde what it is worth in ready Money.*

This is the *Inverſe* of the laſt: for here, if you apply that extent backwards from the number propounded, ſo many times as there are years in the queſtion, you ſhall have your deſire.

Example, 402 *l.* 2 *s.* being due at the end of five years yet to come, I deſire to know how much that ſumme is worth in ready money according to the rate of 8 *l. per centum*: Extend the Compaſſes from 100 to 108, as before: And then, if you apply that extent five times downwards from 402. 1, the movable point will at laſt fall upon 273 *l.* the value of 402. 1, in ready money.

Probl. 3. *A yearly rent or Annuity being forborn a certain number of years, to find what the arrearages thereof will amount unto according to any rate propounded.*

Firſt diſcover the Principal that anſwers to the Rent or Annuity in queſtion, then find unto what ſumme that Principal will be augmented (according to the given rate) at the end of the term propounded: This done, if you ſubtract the ſame Principall out of that

ſumme, the remainder is the ſumme of the arrearages you look for.

Example, a Rent or Annuity of 12 *l. per annum* being forborn 16 years, what will the arrearages thereof amount unto, they being conceived to increaſe (as they grow due) after the rate of 8 *l. per centum?* Here firſt, to find the Principal that anſwers to 12 *l.* ſay thus; If 8 *l.* hath 100 *l.* for his Principal, what ought 12 *l.* to have for his? the anſwer will be (by the 4 *Probleme* of the 4 Chapter) 150 *l.* Having thus diſcovered the Principal of 12 *l. viz.* 150 *l.* I find (by the firſt *Probleme* of this Chapter) that the ſame 150 *l.* being forborn 16 years will amount (after the rate of 8 *l. per centum*) to 513.9, that is 513 *l.* 18 *s.* Now therefore if I deduct 150 *l.* (the correſpondent Principal to the Annuity given) out of 513. *l.* 18 *s.* the remainder, *viz.* 363 *l.* 18 *s.* is the ſumme of the arrearages required.

Probl. 4. *A yearly rent or Annuity being propounded, to find what it is worth in ready money.*

Firſt find what the arrearages thereof amount unto at the end of the term propounded, and then what thoſe arrearages are worth in ready money, which ſhall likewiſe be the required price or value of the Rent or Annuity propounded.

Example, What may a man which is deſirous to lay out his money after the rate of 8 *l.*

per

per centum afford to give for a Leaſe of 12 *l.* *per annum*, that hath yet 16 years in being? I find (by the laſt *Probleme*) that the arrearages of 12 *l. per annum*, being forborn 16 years, amount then unto 363 *l.* 18 *s.* or 363.9, and I find likewiſe (by the ſecond *Probleme* aforegoing) that the ſame 363 *l* 18 *s.* is worth in preſent money 106. 2. or (which is all one) 106 *l.* 4 *s.* I conclude therefore that the value of the Leaſe propounded (at the rate of 8 *l. per centum*) is 106 *l.* 4 *s.*

Here, when the term of the Annuity begins not preſently, but after certain years to come, find what the arrearages forborn for all that time are worth in ready money.

So in the *example* laſt premiſed, if the Annuity of 16 years were not to begin till after the expiration of 5 years, in this caſe you are to enquire what the arrearages (*viz.* 363 *l.* 18 *s.*) being forborn 21 years, are worth in ready money, which you ſhall likewiſe find (by the ſecond *Probleme* before cited) to be 72. 3, which being reduced is 72 *l.* 6 *s.* the value of the Leaſe required.

Probl. 5. *A ſumme of Money being propounded, to find what Annuitie (to continue any number of years, and according to any rate given) that ſumme will buy.*

Take any Annuity at pleaſure, then find

the value of that Annuity in ready money: This done, the *proportion* will be as followeth:

As the value found is to the Annuity taken; so is the summe given to the Annuity required.

Example, What Annuity (to continue 16 years) will 1205 *l.* deserve, so that the purchaser may gain after the rate of 8 *l. per centum?* Here first I take 12 *l. per annum* to continue 16 years, and find the value thereof in ready money (by the last *Probleme*) to be 106. 2, or 106 *l.* 4 *s.* I say therefore,

If 106 2, give 12 *l. per annum*,

What will 1205 *l.* yield? *Facit* 171. 4 *per annum*, which being reduced is 171 *l.* 8 *s.* I conclude therefore, that 171 *l.* 8 *s.* is the Annuity (to endure 16 years) which 1205 *l.* doth deserve, after the rate of 8 *l. per centum.*

Deo Laus.

FINIS.

The Use of A Gauge-Rod, *by the same* AUTHOR.

THe *Gauge-Rod* being three foot in length, hath four Scales described upon it: The first is an ordinary scale of *Inches* numbred by the figures, 1, 2, 3, 4, &c. to 36, and every *Inch* divided into ten parts, after the usual manner, which ten parts are hereafter (for distinction sake) more particularly called *Tenths*. The second (if you turn the Rod towards you) is another scale of equal parts, thus framed, *viz.* by dividing each seven Inches into ten equal parts, untill that whole Scale be throughout so divided; And (that done) it seems to be nothing else, but another Scale of *Inches* of a lesser volume, and without figures; And each of those little *Inches* is again subdivided into lesser parts, in like sort as that other Scale of *Inches* is subdivided: The third (still turning the Rod towards you, as before) is a Scale of *Wine-measure*, first divided into great parts, representing *Wine-gallons*, and distinguished by the larger figures, 1, 2, 3. & 4, set crosse the Rod, and then each of those

great parts (or *Wine-gallons*) are ſubdivided into ten leſſer parts, repreſenting the decimal parts of a *Wine-gallon*, and each of thoſe parts again ſubdivided into as many parts as quantity will permit : The fourth and laſt is a Scale of *Ale-meaſure*, divided into *Ale-gallons*, and afterwards ſubdivided into decimal parts of an *Ale-gallon*, as that of *Wine-meaſure*.

When the Content of a Veſſel is required by the help of this *Rod*, proceed thus :

1. *Meaſure the length of the Veſſel by the Scale of Inches, to the end you may know how many Inches it contains in length from head to head.*

2. *Place the lower end of the Rod (I mean, that end thereof, from which you begin to account the diviſions deſcribed thereupon) at the lower ſide of the head of the Veſſel, within the rimm thereof, cloſe to the head ; then (applying the Rod to the uppermoſt part of the head) move the braſſe Curſor or Feroll, placed next that end of the Rod, ſo high or low, that the uppermoſt end of that Feroll may touch the inſide of the uppermoſt part of the Veſſels rimme, in ſuch ſort that the ſpace comprehended betwixt the lower end of the Rod, and the uppermoſt part of that Feroll may contain the (* Diameter *or) largeſt breadth of the Veſſel at the head.*

3. *Having let down the Rod into the Veſſel*

at

at the Bongue so far, that the lower end thereof may rest upon the lower side of the Vessel, and may stand (perpendicularly, viz) *as upright as may be in the Vessel, fit and justifie the lower end of the other Feroll with the inside of the Vessel at the Bongue.*

4. *Then taking out the Rod, observe and count upon the Scale of Inches the tenths that you find comprehended betwixt the Ferolls, and having counted as many tenths upon the other Scale of lesser Inches from the lower Feroll towards the uppermost, remove the uppermost Feroll towards the lower, untill the lower or inward end thereof may cut the tenths so last counted.*

5. *All this performed, the lower or inward end of the uppermost feroll sheweth how much each inch of the vessells length contains Gallons, and parts of a Gallon, that is to say, upon the scale of* Wine-measure, *the Gallons, and parts, according to that measure, and upon the scale of* Ale-measure, *the like according to that: And therefore if you multiply the Gallons, and parts so found, by the number of Inches contained in the length of the vessell, the result or product will give you the content you look for.*

Example. Admit the Vessel propounded happens to be 32 Inches long, and the uppermost end of the lower Feroll to cut the Scale of Inches at 21 Inches and a half, being the (*Diameter*, or) breadth of the head, and the

lower end of the other Feroll to cut the same Seale at 24 Inches and a half, being the (*Diameter*, or) breadth at the Bongue: In this case, I find upon the Scale of Inches 30 *tenths* to be comprehended upon that scale, betwixt the two Ferolls; and therefore counting 30 *tenths* upon the scale of little Inches from the lower Feroll towards the other, if unto that point I bring down the lower end of the uppermost Ferol, that end upon the scale of *Wine-measure* will cut 1 Gallon, 907 parts of a gallon: Now therefore to find the content of the Vessel in *Wine-measure* (the length of the Vessel being 32 Inches) I multiply 1.907 by 32, and the product is 61.024. that is 61 gallons, and 024 parts of a gallon in *Wine-measure*.

The same direction servs for the due finding out of the Content of a Vessel according to *Ale-measure*, if instead of the Scale of *Wine-measure* you use that of *Ale-measure*: And so in the same case, that Vessel in *Ale-measure* will contain 49 Gallons: For the lower end of the uppermost Ferol cuts one Gallon, 532 parts of a Gallon, which being cast up as in the case of *Wine-measure*, the result will be 49 Gallons, 024 parts of a Gallon.

An

An Appendix Concerning the Measuring OR Gaugeing of Vessel.

By J. NEWTON.

PROP. I.

To find the Content of a Vessel in Cubic Inches.

THat the Content of a Vessel may be known, the form or figure of the Vessel must be agreed on; This few or none deny to be in the form of a perfect Sphæroid, with the two ends equally cut off: Now a Sphære or Sphæroid containeth two third parts of a Cylinder, having the same length and thickness; and therefore the solid convexitie between two Cylinders, one within the Sphæroid, and the other without touching it, and having the same center and height is equal to two third parts of the difference of those two Cylinders, as our English *Archimedes* Mr. *Henry Briggs* hath fully enough shewed in his *Arithmetica Logarithmica*, from these grounds and principles.

The

The Diameters of a Vessel at the Head and Bongue, with the length thereof given in feet or Inches, the Content in the same measure may be readily enough found in this manner as Mr. *William Oughtred* hath shewed in the 9 *Chap.* of his *Circles of Proportion.*

1. *By the Diameters find the superficial Content of the Circles in feet or Inches.*

2. *Then add together two third parts of the superficial Content of the greater Circle, and one third part of the less.*

Lastly, *Multiply the aggregate by the length, the product shall be the content of the Vessel inquired.*

EXAMPLE.

Let the Diameter of a Vessel
At the Bongue be 32 } Inches.
At the Head be 18 }

And let the length of the Vessel be 40 Inches.

The superficial Content of a Circle, whose Diameter is 32 Inches, by the 35 *Probl.* of the 5 *Chap.* is 804. 249. For,

As 1 is to .7854, so is the square of 32 to 804. 249, the superficial Content in Inches.

In like manner the superficial Content of a Circle, whose Diameter is 18 Inches, is 254. 469.

Two thirds of 804. 249, the greater is 536. 166.

One

One third of 254.469, the lesser is 84.823.
The Summe is 620.989.
Which being multiplied by the length 40.
The Product 24839.560.
Is the Content of that Vessel in Inches.

Prop. 2.

To find the Content of a Vessel in Wine-gallons.

THe Content of a Vessel in Cubic Inches being thus found, to find the Content of the same Vessel in Gallons cannot be difficult, the Content of a Gallon in Cubic Inches being also given.

And our English Gallon is understood to be either in Ale or Wine-measure, and our Wine-gallon is without much contradiction supposed to contain 231 Cubic Inches. If therefore it be required, to reduce the Content of a Vessel given in Cubic Inches, to the Content thereof in Wine-gallons; Divide the Content given by 231, the number of Cubic Inches in a Wine-gallon, the quotient is the Content inquired.

Example, Let 24839.56 the Content of a Vessel in Cubic Inches be divided by 231, the quotient

quotient 107. 52, is the Content thereof in Gallons.

This method of Gaugeing, is as exact as that propounded by our Author of this *Rule of Proportion*, and may be made altogether as easie, if a Table or line be made, shewing the superficies of a Circle in Gallons and parts of a Gallon to every inch of a Diameter, or rather let the third part of that Content be set in the Table for the Diameter at the Head, and two thirds for the Diameter at the Bongue; yet to the intent that this method of Gaugeing may be compared with that in the 10 Chapter, and also with that Gaugeing Rod of the same Authors now annexed to this Treatise, in the Table following there is set down the whole superficial Content of a Circle in Wine-gallons, as also one third part, and two third parts of the same Content, whose use we will explain in the Examples following.

A TABLE

A TABLE for the Gaugeing of Wine-Vessels.

Inches	whole Content	Differ.	Head one third	Differ.	Bung two thirds	Differ.
11	0.411	72	0.137	0.024	0.274	48
12	0.489	79	0.163	0.026	0.326	52
13	0.576	87	0.191	0.029	0.385	59
14	0.666	90	0.222	30	0.444	60
15	0.765	99	0.255	33	0.510	66
16	0.870	105	0.290	35	0.580	70
17	0.983	113	0.328	37	0.657	74
18	1.102	119	0.367	39	0.734	79
19	1.227	125	0.409	42	0.818	84
20	1.360	133	0.453	44	0.906	88
21	1.499	139	0.500	46	1.000	92
22	1.646	147	0.548	49	1.097	99
23	1.799	152	0.599	51	1.199	102
24	1.958	160	0.652	53	1.305	106
25	2.125	167	0.708	56	1.416	112
26	2.298	173	0.766	57	1.532	114
27	2.479	181	0.826	60	1.652	120
28	2.666	187	0.888	62	1.777	124
29	2.859	193	0.953	64	1.906	128
30	3.060	201	1.020	67	2.040	134
31	3.267	207	1.090	69	2.180	139
32	3.482	215	1.160	71	2.321	142
33	3.703	221	1.234	73	2.468	146
34	3.930	227	1.310	75	2.620	150
35	4.165	235	1.388	78	2.776	156

Inches	whole Content	Differ:	Head one third	Differ:	Bung two thirds	Differ:
36	4.406	241	1.468	80	2.936	160
37	4.655	249	1.551	83	3.102	166
38	4.910	255	1.636	85	3.272	170
39	5.171	261	1.723	87	3.447	174
40	5.440	269	1.813	89	3.625	179
41	5.715	275	1.904	92	3.809	184
42	5 998	283	2.000	94	4 000	188
43	6.287	289	2.095	96	4.190	192
44	6.582	295	2.194	98	4.388	196
45	6.885	303	2.294	101	4.588	202
46	7.194	309	2.396	103	4.795	206
47	7.511	317	2.503	105	5.007	210
48	7.834	323	2.610	107	5.220	214
49	8.163	329	2.721	109	5.442	219
50	8.500	337	2.832	112	5.665	224
51	8.843	343	2.947	114	5.895	228
52	9.194	351	3.064	117	6 127	234
53	9.551	357	3.275	119	6 550	239
54	9.914	363	3.304	121	6 609	243
55	10 285	371	3.428	123	6.856	247
56	10.662	377	3.554	125	7.108	251
57	11.046	384	3.682	128	7.364	257
58	11.437	393	3.812	131	7.624	263
59	11.835	398	3.945	132	7.890	265
60	12.240	405	4.079	135	8.157	271

I. EXAM-

1. EXAMPLE.

Let the Diameter of a Veſſel at the Bongue be 32 Inches, and the Diameter at the Head 18, then will the Equated Diameter according to Mr. *Wingate* be 27. 80, and the Wine-gallons and parts anſwering to 27 Inches 80 parts of an Inch by this Table are 2. 628, which being multiplied by 40 the length of the Veſſel, the Product 105. 12 : whereas the Content of the ſame Veſſel according to Mr. *Oughtred* was before found to be 107.52, that is two Gallons and a half more *ferè*.

2. EXAMPLE.

Let the Diameter of a Veſſel at the Bongue be 245, and the Diameter at the Head 21. 5 Inches : Or,

The third of the Circle according to the greater Diameter 24. 5, is	1.361
One third according to the leſs 21.5, is	0.524
Their Summe is	1. 885
The length of the Veſſel	32
The Product	60.320

Whereas the Content of the ſame Veſſel according to Mr. *Wingate* was before found to be 61. 024, that is 704 parts of a Gallon more then by the other way of working.

PROP. 3.

Prop. 3.

To find the Content of a Vessell in Ale-Gallons.

THe Diameter of a vessell at the Head and Bung with the length being given, the Content thereof in Ale-gallons may be found by the former rules, if the quantity of an Ale-Gallon in Cubic-Inches be first determined, but of this there hath of late been much dispute, and therefore to give the Reader and those that are concerned in the quantity of this measure all possible satisfaction, I wil here set down what the Law of our Land hath formerly and doth still in this case command to be observed.

In the 51 *Hen.* 3. It was Enacted by the Lords and Commons in Parlament then assembled, that the Kings measure should bee made, so that an English peny which is called the *Sterling*, round without clipping, should weigh 32 grains of wheat well dried and gathered out of the middle of the ear, and 20 pence make an Ounce, and 12 ounces make a pound Troy, 8 pound a Gallon of Wine, and 8 Gallons a Bushell of London, which is the eight part of a Quarter, and according to these directions was the standard measures at the Exchequer then made, and others from them

them exactly agreeing with them commanded to be used and no other throughout the Realm of England, but what through the ignorance of artificers or rather the deceitfulnesse of those that imploied them, Standards were made which did not exactly agree with the Standard in the Exchequer, and therefore *Anno* 31 *Edward* 1. the former Law was confirmed, the Standards in all places of this Nation commanded to be broken, and new made according to the former direction ; So likewise 12 *Hen.*7.*cap.*5. and *Anno* 23 *Hen.*8.*cap.*4. Every Artificer of the craft or mystery of Coopers shall make the Vessels for Beer and Ale, so that every Barrel for Beer shall contain 36 Gallons, every Kilderkin for Beer 18 Gallons, and every Firkin for Beer 9 Gallons of the Kings Standard Gallon. And that every Barrel for Ale shall contain 32 Gallons, every Kilderkin for Ale 16 Gallons, and every Firkin for Ale 8 Gallons of the Kings Standard Gallon ; And that no Cooper shall make any other Vessel for Beer or Ale to be sold within this Realm, of any greater or lesser number of Gallons then is above-said ; unlesse he shall cause to be marked upon every such Vessel the true and certain number of as many Gallons as every such Vessel shall contain, to the intent that every person may know the Content thereof; and these Statutes do stil stand in force

and unrepealed, nay have from time to time been ſtill ratified and confirmed; and therefore the Gallon of the Exchequer is the onely Standard Gallon of this Nation; what other Gallons are in uſe amongſt us, are either by privilege, as thoſe of the Company of Vintners, and *Winchester* meaſure; or elſe by deceitful and corrupt practice; amongſt which, that now remaining in Coopers-hall may well be reckoned, ſince they are not onely commanded to make their Veſſel by the Kings Standard, but to mark upon them the true Content according to that Standard, if at any time they make them otherwiſe: Beſides the Quarts, by which your Ale-houſe keepers are bound to ſell their Ale as well as Beer do exactly agree with that Standard, as I can and do teſtifie upon mine own experience.

So that there is but little, if any thing at all, to be ſaid in defence of the Coopers-gallon; and yet the Brewers have no great reaſon to complain of any wrong or injuſtice done them, though they pay Exciſe according to the Standard in the Exchequer, and ſell by the Standard in Coopers-hall, becauſe the Commiſſioners for Exciſe may and do, by virtue of an Act of Parlament made 1649, Chap. 50. Article 34, give ſuch allowance to the Brewers for Filling & Leakage, as reaſonably they can demand; and this Allowance is no

waies

waies given to the Ale-houſe-keepers, who pay the Exciſe, that I can be informed of, but by the over-meaſure which they have in the quantity of their Barrel; & unleſs that ſuch allowance be, they will pay Exciſe for 36 Gallons, according to the Standard in the Exchequer, and from the Brewer receive in ſaleable drink little more then 34 Gallons according to the ſame meaſure: I can therefore ſee no reaſon for meaſuring our Veſſel by any other Gallon then this of the Exchequer, to which we are obliged by Law: And according to this I have therefore framed my Table and Examples following, taking the true Content thereof to be 272. 25 Cubic Inches, as Mr. *Oughtred* in his Circles of Proportion hath affirmed; or rather 272 Cubic Inches juſt, as others have lately found by meaſuring of that Gallon in the Exchequer, which is the preſent Standard of this Nation.

A TABLE for the Gaugeing of Beer and Ale-Veſſel.

Inches	whole Content	Differ:	Head one third	Differ:	Bung two thirds	Differ:
11	0.350	60	0.116	20	0.232	40
12	0.416	66	0.135	22	0.270	44
13	0.488	72	0.166	24	0.333	48
14	0.566	78	0.188	26	0.377	52
15	0.650	84	0.216	28	0.433	56
16	0.739	89	0.246	29	0 492	58
17	0 834	95	0.278	31	0.556	62
18	0 936	101	0.312	33	0.624	66
19	1.042	107	0.347	35	0.694	70
20	1.155	113	0.385	37	0.770	74
21	1.273	118	0.424	39	0.884	78
22	1.398	124	0.466	41	0.932	82
23	1.527	130	0.509	43	1.018	86
24	1.663	136	0.554	45	1 108	90
25	1.805	142	0.601	47	1 202	94
26	1.952	147	0.650	49	1.300	98
27	2.105	153	0.701	51	1.402	102
28	2.264	158	0.754	52	1.508	104
29	2.428	165	0.809	55	1.618	110
30	2.599	170	0.866	57	1.733	114
31	2.775	176	0.925	58	1.851	107
32	2.957	182	0.985	60	1.971	120
33	3.144	187	1.048	62	2.097	124
34	3.338	194	1.112	64	2.224	128
35	3.538	199	1.179	66	2.359	132

Inches	whole Content	Differ:	Head one third	Differ:	Bung two thirds	Differ:
36	3.742	205	1.247	68	2.495	136
37	3.953	211	1.317	70	2.634	140
38	4.170	217	1.390	72	2.780	144
39	4.392	222	1.464	74	2.928	148
40	4.620	228	1.540	76	3 080	152
41	4.854	234	1.618	78	3.236	156
42	5.094	240	1.698	80	3.397	160
43	5.339	245	1.779	81	3.558	162
44	5.590	251	1.863	83	3.726	166
45	5.847	252	1.949	85	3.899	170
46	6.110	263	2.033	87	4.066	174
47	6.379	269	2.128	89	4.257	178
48	6 653	274	2.217	91	4.434	182
49	6.933	280	2.311	93	4 622	186
50	7.219	286	2 406	95	4 812	190
51	7.510	291	2.503	97	5.006	194
52	7.808	298	2.602	99	5.204	198
53	8.111	303	2.703	101	5.406	202
54	8 420	309	2.806	103	5 612	206
55	8.735	315	2.911	105	5.822	210
56	9.055	320	3.018	106	6 036	212
57	9.381	326	3.127	108	6. 254	216
58	9.714	333	3.238	111	6.476	222
59	10.051	337	3.350	112	6.700	224
60	10.395	344	3 465	114	6.930	228

1. EXAM-

1. EXAMPLE.

Suppose a Vessell whose Diameter at the Bung is in Inches. 32
and at the Head in Inches. 18
The length in Inches. 40

The numbers in this table answering to

32 Inches at the Bung is	1. 971
18 Inches at the Head is	0. 312
There summe	2. 283
Being multiplied by the length	40
The content is	91. 320

Or according to M. *Wingate* the Æquitated Diameter is Inches. 27.8, and the whole superficiall content of a Circle in Ale-Gallons answering to that Diameter is Gallons 2. 231, which being multiplied by 40 the length of the Vessel, the product 89.240 is the content thereof, 2 Gallons. 08 parts of a Gallon less, then by the former way of working.

2. EXAMPLE.

Let the Diameter of a Vessell at the Bung be Inches. 24. 5
at the Head be Inches. 21, 5
The length in Inches. 32

The number in this table answering to

24. 5 Inches at the *Bung* is	1. 155
21. 5 Inches at the Head is	0. 444

Their

Their ſumme	1.599
Being multiplied by the length	32
The content in Gallons is	51.168

Or according to M. *Wingate* the equated Diameter is Inches 23.6, and the whole Superficiall content of a circle in Ale-Gallons anſwering to that Diameter is Gallons. 1.609, which being multiplyed by 32 the length of the Veſſel, the product 51.488 is the content thereof. 32 parts of a Gallon more then by the former way of working.

Theſe Examples we deem ſufficient to illuſtrate the uſe of the Table, which being made according to the two beſt and moſt generall received ways of Gauging, I leave it to the diſcretion of the Ingenious Artiſt to take that which ſuteth beſt with his own liking.

FINIS.

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